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Review

Kinetic Treatment of Exact Solution of Thermal Radiation Field Affected on a Rarefied Gas in Steady State.

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The studying the influence of a thermal radiation field upon a rarefied neutral gas is introduced. We insert the radiation field effect in the force term of the Boltzmann equation. In a frame co-moving with the fluid, analytically the BGK (Bhatnager- Gross- Krook) model kinetic equation is applied. The problem is studied using Liu-Lees model. We apply the moment method using the linearized form of thermal radiation field to follow the behavior of the macroscopic properties of the gas such as the temperature and concentration. They are substituted into the corresponding two stream Maxwellian distribution functions permitting to investigate the non-equilibrium thermodynamic properties of the system (gas + heated plate). The entropy, entropy flux, entropy production, thermodynamic forces, kinetic coefficients are obtained. We verify the celebrated Onsager's reciprocity relation for the system. The ratios between the different contributions of the internal energy changes based upon the total derivatives of the extensive parameters are estimated via the Gibbs formula. The results are applied to the Argon gas for definite radiation field intensity corresponding to the plate temperature. Graphics illustrating the calculated variables are drawn to predict their behavior and the results are discussed.

Keywords: Rarefied gas dynamics; Heat transfer; Thermal radiation field; Boltzmann kinetic equation; BGK model; Entropy production; Onsager's reciprocity relation; Gibb's formula.

INTRODUCTION

The radiative processes play a major role in the thermodynamics of the Earth system. For this purpose, researchers have used simple blackbody (BB) type planetary models to estimate theoretically planetary entropy production rates. The analysis of simple radiative models of the Earth system provides insight into its thermodynamic behavior even though it is complex. From a thermodynamic perspective thermal radiation (TR) exchange, i.e., incoming sunlight and outgoing TR, is the only significant form of energy transfer between the Earth and the universe. Further, processes such as absorption and emission dominate planetary entropy production, and the non-uniform absorption of solar radiation (SR) on the Earth causes circulation of the atmosphere and oceans (Sean, 2007), (Wright et al., 2000). They have analyzed simple blackbody type radiative models to investigate the thermodynamic behavior of the Earth system and to estimate planetary temperature and entropy production rates (Aoki, 1982; Stephens and O'Brien,1993; Weiss, 1996). It is more accurate to model the Earth system as a gray-body because absorption of sunlight and emission of TR are substantially less than that of a blackbody (Wright et al., 2000).

Non-equilibrium steady states are usually obtained in the laboratory by application of appropriate boundary conditions. From a theoretical, as well as a computer simulation, point of view, an example of the steady state related to our paper is the work conducted Ibrahim and Hady (Ibrahim and Hady, 1990). They discussed the effect of the thermal radiation on the steady laminar plane flow over an isolated horizontal flat plate .Within the framework of first-order boundary-layer they consider the fluid as a gray absorbing–emitting but not scattering medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is also useful to consider steady states driven out of equilibrium by the action of external forces. In the limit of small forces, the response of the system, as measured by the presence of hydrodynamic gradients and fluxes, is linear. In general, the strength of the forces provides a parameter measuring the departure of the system from equilibrium. Some examples of non-equilibrium steady states generated by external forces are the centrifugal force field (Abourabia and El-Malky, 2007) , the homogeneous heat conductivity (Garzó and Santos, 1991) and color conductivity (Garzó and Santos, 1991) problems.

The gas influenced by a thermal radiation flied was investigated by some authors in both linearized and non-linearized radiation heat flux formula (Kuznetsov and Sheremet, 2009 : Abo-Eldahab, 2001 : Perdikis and Raptis, 1996 : Bernard and Eduard, 2006). Usually, they consider that the gas is dense, so that it obeys Navier-Stokes equations. We investigate the situation when a nonlinear thermal radiation force acting on a rarefied neutral gas within the framework of the molecular gas dynamics and the kinetic Boltzmann equation which gives us more accurate descriptions of physical reality than some others equations do. The kinetic equation of gas flows based on the Boltzmann equation has obvious peculiarities in comparison with the macroscopic description by means of Navier-Stokes equations. In principle, the kinetic equation posses the important advantages as it is a tool for considering non-equilibrium thermodynamics, nonlinear processes, i.e. strong deviation from equilibrium (Aristov, 2001), (Alferd and Beylich, 2000), while the Navier-Stokes equations is valid only in a small deviations of equilibrium thermodynamics. The second advantage, the kinetic equation could be the origin for obtaining simple models and solutions for the description of very complicated physical situations, such as gas motion with a very large velocity (in respect with the thermal speed) as it is valid for all range of Mach numbers, while the Chapman-Enskog expansion (and consequently the Navier-Stokes equations) is not adequate to describe turbulent flows (Aristov, 2001). The third one, Boltzmann equation is valid for studying the flows in all range of Knudsen number, i.e. the slip, transition, continuum, and free molecular regimes (Zhdanov and Shulepov, 1975) while the Chapman-Enskog method proved the validity of the Navier-Stokes equations as a limit at small Knudsen numbers in continuum regime. However, there were some restrictions on macroscopic equations in considering rarefied gas regimes. The attempts to construct rarefied gas dynamics similar to the macroscopic equations of hydrodynamics without using the multidimensional phase space were unsuccessful. The necessity of studying the Boltzmann kinetic equation itself was obvious. The fourth, for weak rarefied medium the solution of the Boltzmann kinetic equation is matched in the boundary layers, where as the solution of the macroscopic equations are out side this layer (Aristov, 2001).

Our aim in this paper is as follows: in section (**2)** to introduce an approach for studying the influence of thermal radiation field on a rarefied neutral gas. For this purpose, we use the Kinetic Boltzmann equation instead of the Navier–Stokes equations, which are satisfied only for the dense gases. We insert the radiation field effect into the term force of the Boltzmann equation as a radiation force. This idea was applied on a steady problem of the half space filled by a neutral gas specified by a flat rested heated plate in a frame co-moving with the gas. We apply this approach, using Liu-Lees model for two stream Maxwellian distribution functions and the moment method to predict the behavior of the macroscopic properties of the gas, such as the temperature and concentration, which in turn are substitute into the corresponding distribution functions. This tackling, in section (**3),** permits us to study the behavior of the equilibrium and non- equilibrium distribution functions for definite plate temperature. The important non-equilibrium thermodynamic properties of the system (gas + heated plate) are calculated. Namely we obtain the entropy, entropy flux, entropy production, thermodynamic forces, kinetic coefficients .We investigate the verification of the celebrated Onsager's reciprocity relation and Onsager's inequality. The ratios between the different contributions of the internal energy change based upon the total derivatives of the extensive parameters are predict via the Gibb's formula. Section **(4)** shows the discussion and conclusions of the results applied to the Argon gas for definite radiation field intensity corresponding to the plate temperature.

The Physical Problem and Mathematical Formulation

Let us assume that the upper half of the space ($y \geq 0$), which is bounded by an infinite immobile flat plate (y=0), is filled with a monatomic neutral dilute gas with a uniform pressure P_s (Zhdanov and Roldughin, 1998), (Zhdanov and Roldugin, 1996) and the plate is heated suddenly to produce heat radiation field. The flow is considered steady, one-dimensional, and compressible. The behavior of the gas is studied under the assumptions that:

(i) The gas molecules are reflected from the plate with full energy accommodation.

(ii) The gas is considered gray absorbing-emitting but not a scattering medium.

(iii) A thermal radiation force is acting from the plate on the gas in the vector form (Mitchell Thomas, 1964), (Nick Kaiser, 2002)

$$\vec{F} = \frac{-4\sigma_s}{3n_s c}\vec{\nabla} T^4(y) \Rightarrow F_y = \frac{-4\sigma_s}{3n_s c}\frac{dT^4(y)}{dy}$$

(1)

For steady motion, the process in the system under study subject to a thermal radiation force F_y can be expressed in terms of the Boltzmann kinetic equation [19-21] in the BGK model written in the form:

$$C_y \frac{\partial f}{\partial y} + \frac{F_y}{m}\frac{\partial f}{\partial C_y} = \frac{(f_0 - f)}{\tau}$$

(2)

where

$$f_0 = \frac{n}{(2\pi RT)^{\frac{3}{2}}}\exp\left[\frac{-C^2}{(2RT)}\right] : C^2 = C_x^2 + C_y{}^2 + C_z^2 .$$

(3)

Lee's moment method [22-28] for the solution of the Boltzmann's equation is employed here. One of the most important advantages of this method is that the surface boundary conditions are easily satisfied. Maxwell converted the Maxwell-Boltzmann equation into an integral equation of transfer, or moment equation, for any quantity Q that is a function only of the molecular velocity. The distribution function used there should be considered as a suitable weighting function, which is not the exact solution of the Maxwell-Boltzmann equation in general. Lees found that the distribution function employed in Maxwell's moment equation must satisfy the following basic requirements: (i) It must have the "two- sided" character that is an essential feature of highly rarefied gas flows.(ii) It must be capable of providing a smooth transition from free molecule flows to the continuum regime.(iii) It should lead to the simplest possible set of differential equations and boundary conditions consistent with conditions (i) and (ii). When the application of heat to a gas causes it to expand, it is thereby rendered rarer than the neighboring parts of the gas; and it tends to form an upward current of the heated gas, which is of course accompanied with a current of the more remote parts of the gas in the opposite direction. The gas is thus made to circulate; fresh portions of gas are brought into the neighborhood of the source of heat, carrying their heat with them into other regions (Maxwell and Rayleigh, 1902). We assume the temperature of the upward going gas particles is T_1 while the temperature of the downward going gas particles is T_2. The corresponding concentrations are n_1 and n_2.Making use of the Liu-Lees model of the two–stream Maxwellian distribution function near the plate suggested by Kashmarov (Wasserstrom et al., 1965), (Shidloveskiy, 1967) in the form:

$$f = \begin{cases} f_1 = \dfrac{n_1}{(2\pi RT_1)^{\frac{3}{2}}}\exp[\dfrac{-C^2}{(2RT_1)}] , For \quad C_y > 0 \\ f_2 = \dfrac{n_2}{(2\pi RT_2)^{\frac{3}{2}}}\exp[\dfrac{-C^2}{(2RT_2)}] , For \quad C_y < 0 \end{cases}$$

(4)

The solution of the Boltzmann equation is extremely difficult, and the velocity distribution function f is not directly of interest to us, in this stage, but the moments of the distribution function are of interest. Therefore, we derive the Maxwell's Moment equations by multiplying the Boltzmann equation by a function of velocity $Q_i(C)$ and integrating over the velocity space. How many and what forms of Q_i will be used is dependent on how many unknown variables need to be determined and how many equations need to be solved. Multiplying equation (2) by some functions of velocity $Q_i = Q_i(C)$, and integrating w. r. t. C taking into consideration the discontinuity of the distribution function caused by the cone of influence (Shidloveskiy, 1967)- (Cercignani, 1988), (Chapman and cowling, 1970) and (Jeans, 1904) showed that the resulting equation can then be written as :

$$\frac{d}{dy}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} Q_i C_y f_2 d\underline{C} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} Q_i C_y f_1 d\underline{C}\right]$$
$$-\frac{(F_y)}{m}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_2 \frac{dQ_i}{dC_y} d\underline{C} + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_1 \frac{dQ_i}{dc_y} d\underline{C}\right] = \tag{5}$$
$$\frac{1}{\tau}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Q_i f_0 d\underline{C} - \left(\int_{-\infty}^{\infty}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f_2 + \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{-\infty}^{\infty} f_1\right) Q_i d\underline{C}\right], \text{where } d\underline{C} = dC_x dC_y dC_z$$

where F_y is the external force defined by equation (1).Equation (5) is called the general equation of transfer or the transfer equation .

We can get the dimensionless forms of the variables by taking:

$$y = \frac{5\sqrt{\pi}}{4}\left(\frac{\mu_S\sqrt{2RT_s}}{n_S T_S}\right)\bar{y}, \vec{C} = \bar{\vec{C}}\sqrt{2RT_s}, f_i = \frac{\bar{f}_i (2\pi RT_s)^{\frac{3}{2}}}{n_s}, i = 0,1,2 \tag{6}$$

$$T_1 = \bar{T}_1 T_s \ , n_1 = \bar{n}_1 n_s , T_2 = \bar{T}_2 T_s \ , n_2 = \bar{n}_2 n_s , \qquad \text{and } dU = \overline{dU}(K_B T_s).$$

For the sake of simplicity henceforth, we drop the dash over the dimensionless variables. It is assumed that the temperature differences within the gas are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding F_y in a Taylor series about T_∞ and neglecting higher-order terms (Perdikis and Raptis, 1996 : Ahmed et al., 2002), (Ouaf, 2005 : Osalusi, 2007), thus $T^4 \cong 4T_\infty{}^3 T - 4T_\infty{}^4$ This implies that,

$$F_y = \frac{-4\sigma_s}{3n_s c}\frac{d}{dy}\left[4T_\infty{}^3 T - 4T_\infty\right] = \frac{-16\sigma_s T_\infty{}^3}{3n_s c}\frac{dT}{dy}$$

Once the expressions for f_0, f_1 and f_2 are introduced, macroscopic quantities such as density, velocity, temperature, etc… can be computed from the appropriate weighted integral of the distribution functions as follows ; Number density (Shidloveskiy, 1967 : Cercignani, 1988):

$$n(y) = \int f(y, C_y)\, d\underline{C} = \frac{(n_1 + n_2)}{2} \quad . \tag{7}$$

Hydrodynamic (bulk) velocity:

$$u(y) = \frac{1}{n}\int C_y\, f(y, C_y)\, d\underline{C} = \frac{\left(n_1\sqrt{T_1} - n_2\sqrt{T_2}\right)}{(n_1 + n_2)}. \tag{8}$$

The temperature:

$$T(y) = \frac{1}{3n}\int C^2 f(y, C_y)\, d\underline{C} = \left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right) . \tag{9}$$

The static pressure normal to the plate:

$$P_{yy} = \int C_y{}^2\ f(y, C_y)\, d\underline{C} = \frac{1}{2}(n_1 T_1 + n_2 T_2). \tag{10}$$

The heat flux component:

$$q_y(y) = \int C_y C^2 f(y, C_y)\, d\underline{C} = \left(n_1 T_1^{\frac{3}{2}} - n_2 T_2^{\frac{3}{2}}\right).$$

In Equation (4) there are four unknown functions T_1, T_2, n_1 and n_2 needed to be determined. Thus we need four moment equations to solve our problem. We take $Q_i = 1, C_y, C^2$ and $\frac{1}{2}C_y C^2$, and substitute Equation (4) into Equation (5). We note that for a neutral gas, this procedure will give rise to continuum conservation equations.

$Q_i = 1$ (continuity), $Q_i = C_y$ (momentum conservation), $Q_i = C^2$ (energy conservation) and $Q_i = \frac{1}{2}C_y C^2$ (energy transport). After dropping the bars we get the following four equations, the conservation of mass

The conservation of mass:

$$\frac{d}{dy}\left[\left(n_1 T_1^{\frac{1}{2}} - n_2 T_2^{\frac{1}{2}}\right)\right] = 0. \tag{11}$$

The conservation of y-momentum:

$$\frac{d}{dy}\left[(n_1 T_1 + n_2 T_2)\right] - N\left(\frac{d}{dy}\left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right)\right)(n_1 + n_2) = 0, \tag{12}$$

The conservation of energy :

$$\frac{d}{dy}\left[\left(n_1 T_1^{\frac{3}{2}} - n_2 T_2^{\frac{3}{2}}\right)\right] + N\left(\frac{d}{dy}\left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right)\right)\left(n_1 T_1^{\frac{1}{2}} - n_2 T_2^{\frac{1}{2}}\right) = 0, \tag{13}$$

The heat flux equation:

$$\frac{5}{4}\frac{d}{dy}\left[\left(n_1 T_1{}^2 + n_2 T_2{}^2\right)\right] + \frac{3}{2}N\left(\frac{d}{dy}\left(\frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}\right)\right)(n_1 T_1 + n_2 T_2) = \left[\frac{-2}{\sqrt{\pi}K_n}\left(n_1 T_1^{\frac{3}{2}} - n_2 T_2^{\frac{3}{2}}\right)\right], \tag{14}$$

where $\tau = \frac{5\sqrt{\pi}}{4}\frac{\mu_s}{n_s T_s}K_n$, (Shakhov, 1969), where K_n is the hydrodynamic Knudsen number defined by :

$$K_n = \frac{\text{Mean free path}}{\text{Hydrodynamic length}} = \frac{\lambda}{\left(\frac{\mu_s}{n_s T_s}V_{Th}\right)},$$

and $N = \frac{16\sigma_s T_\infty{}^3}{3n_s cmR}$ is a non-dimensional constant.

We intend to solve equations (11 through 14) to estimate the four unknowns T_1, T_2, n_1 and n_2. By integrating equation (11) w. r. t. y and using equation (8) we get :

$$\left(n_1 T_1^{\frac{1}{2}} - n_2 T_2^{\frac{1}{2}}\right) = C_1 = n\ u(y)$$, C_1 is the integration constant . (15)

We shall study the problem in a coordinate system of the phase space in which $u(y)$ is located at the origin, therefore $C_1 = 0$. Therefore

$$n_1\sqrt{T_1} = n_2\sqrt{T_2}. \tag{16}$$

Equations (13) and (16) then yield

$$\frac{d}{dy}\left(n_1 T_1^{\frac{3}{2}} - n_2 T_2^{\frac{3}{2}}\right) = \frac{d}{dy}\left(n_2\sqrt{T_2}\,(T_1 - T_2)\right) = 0 \quad . \tag{17}$$

Integrating equation (17) w. r. t. y, we obtain after

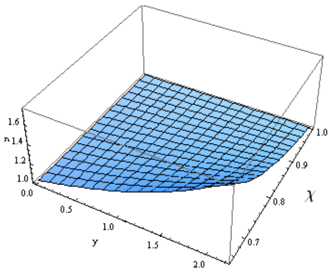


Figure (1): Concentrations n vs. y and χ.

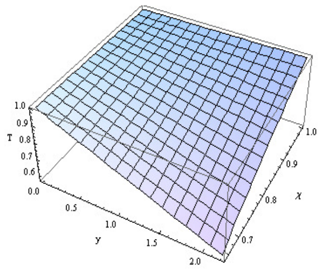


Figure (2): Temperature T vs. y and χ .

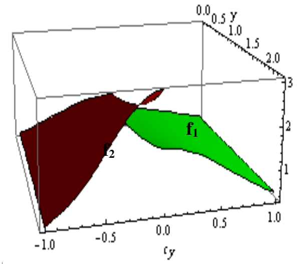


Figure (3): Distribution function f as $\chi=0.65$.

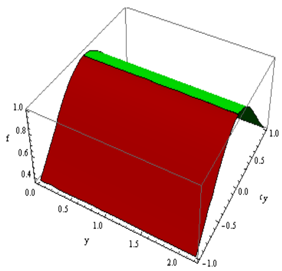


Figure (4): Distribution function f as $\chi=1.0$.

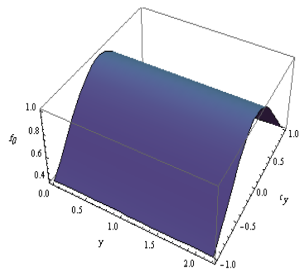


Figure (5): The equilibrium distribution function f_0

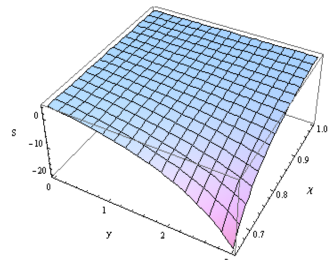


Figure (6): Entropy S vs. y and χ

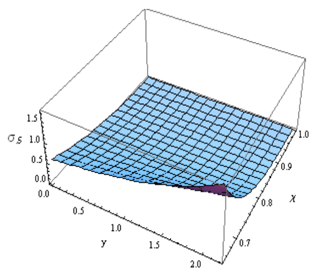


Figure (7): Entropy production σ_s vs. y and χ.

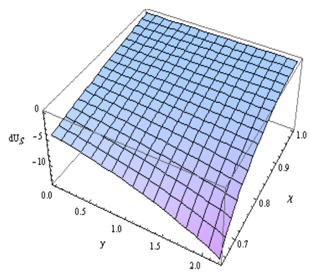


Figure (8): dU_S vs. y and χ.

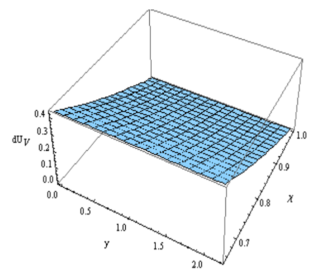


Figure (9): dU_v vs. y and χ.

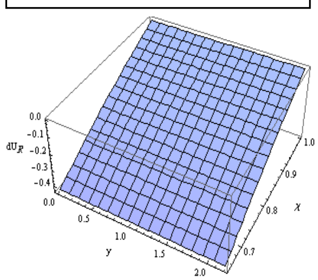
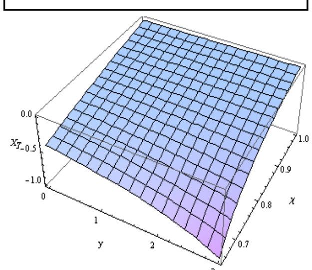
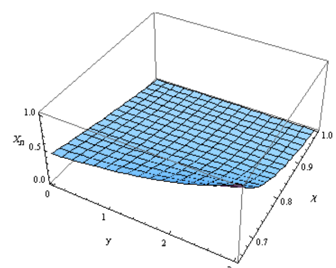


Figure (10): dU_R vs. y and χ.



Figure(11):Thermodynamic force X_T vs. y and χ.



Figure(12):Thermodynamic force X_n vs. y and χ.

factorization:

$$\left(n_2\sqrt{T_2}\left(\sqrt{T_1}+\sqrt{T_2}\right)\left(\sqrt{T_1}-\sqrt{T_2}\right)\right) = \omega_1\omega_2 = C_2, \qquad (18)$$

where

$$\omega_1 = n_2\sqrt{T_2}\left(\sqrt{T_1}+\sqrt{T_2}\right), \quad \omega_2 = \left(\sqrt{T_1}-\sqrt{T_2}\right), \qquad (19)$$

and C_2 is the integration constant, this comes from the assumption of the pressure uniformity since $P_{yy} = \frac{n_2}{2}\sqrt{T_2}\left(\sqrt{T_1}+\sqrt{T_2}\right) = \frac{\omega_1}{2}$ is independent of y (Zhdanov and Roldughin, 1998) : - (Zhdanov and

Roldugin, 1996), this implies that ω_2 is a constant as well.
With the help of equation (16) we can put

$$\theta(y) = n_2\sqrt{T_2} = n_1\sqrt{T_1}.$$

(20)
From equations (19) and (20) we can obtain by simple algebraic steps that :

$$T_1 = \frac{(\omega_1+\omega_2\theta)^2}{4\theta^2},\ T_2 = \frac{(\omega_1-\omega_2\theta)^2}{4\theta^2},\ n_1 = \frac{2\theta^2}{(\omega_1+\omega_2\theta)}\ \text{and}\ n_2 = \frac{2\theta^2}{(\omega_1-\omega_2\theta)}. \quad (21)$$

Integrating equation (14) w. r. t. y and with the help of equations (12,17) we get

$$\frac{5}{4}\left[\left(n_1T_1^2+n_2T_2^2\right)\right]+\frac{3N}{4}\frac{\left(n_1T_1+n_2T_2\right)^2}{(n_1+n_2)} = -\frac{2}{\sqrt{\pi}}\frac{\omega_1\omega_2 y}{K_n}+\omega_3 \quad (22)$$

where ω_3 is the integration constant.
Substituting from equations (21) into equation (22) yields by solving we get :

$$\theta(y) = \frac{\pm\sqrt{(-5-3N)(Kn\sqrt{\pi}\omega_1^3)}}{\sqrt{32y\,\omega_1\omega_2+(15-3N)(Kn\sqrt{\pi}\omega_1\omega_2^2)-16\sqrt{\pi}Kn\omega_3}} \quad (23)$$

we keep into consideration the positive root which preserves the positive signs of both temperature and concentration.

The values of the constants ω_1, ω_2 and ω_3 can be obtained under the boundary conditions (at y =0)

$$\frac{\left(n_1(0)+n_2(0)\right)}{2} = 1 ,$$

(24)

$$\left(\frac{n_1(0)T_1(0)+n_2(0)T_2(0)}{n_1(0)+n_2(0)}\right) = 1 ,$$

(25)

$$\left(n_1(0)T_1(0)^{\frac{1}{2}} - n_2(0)T_2(0)^{\frac{1}{2}}\right) = 0 .$$

(26)
The temperature of the incident particles is assumed to be T_2 while the temperature of the reflected particles from the plate is the temperature T_1, they are related such that (Jeans, 1904) (Chapman and cowling, 1970)

$$T_2(0) = \chi\ T_1(0)\ :\ 0 < \chi \le 1,$$

(27)
where χ is the temperature ratio between the temperature of the downward going gas particles T_2 (incident with the gas temperature) and the upward going gas particles T_1 after reflection from the heated plate surface (with the plate temperature) (Mitchell Thomas, 1964), (Shidloveskiy, 1967) since we suppose that the energy accommodation coefficient equals to unity.

The parameter χ can take arbitrary positive value less than unity to guaranty that the plate is hotter than the gas. We can obtain by solving the algebraic system of equations (20) that
We can obtain by solving the algebraic system of equations (Wasserstrom et al., 1965 : Chou et al., 1966 : El-Sakka et al., 1985 : Khater and El-Sharif, 1988) that

$$n_1(0) = \left(\frac{2\sqrt{\chi}}{1+\sqrt{\chi}}\right), n_2(0) = \left(\frac{2}{1+\sqrt{\chi}}\right), T_1(0) = \left(\frac{(1+\sqrt{\chi})}{\chi+\sqrt{\chi}}\right)\ \text{and}\ T_2(0) = \left(\frac{\chi(1+\sqrt{\chi})}{\chi+\sqrt{\chi}}\right). \quad (28)$$

The above four quantities represent the boundary conditions.
By substituting from (28) into (19) we get

$$\omega_1 = 2, \omega_2 = \frac{1}{\chi^{\frac{1}{4}}} - \chi^{\frac{1}{4}}. \quad (29)$$

By substituting from (28) and (29) into (22) we get

$$\omega_3 = \frac{5-5\sqrt{\chi}+3N\sqrt{\chi}+5\chi}{2\sqrt{\chi}} \quad (30)$$

Equations (21,23,29,30) give the exact values of n_1, n_2, T_1 and T_2 therefore substituting the obtained four quantities into the two stream Maxiwallian distribution functions

$$\begin{cases} f_1 = \dfrac{n_1}{T_1^{\frac{3}{2}}}\exp[\dfrac{-C^2}{T_1}]\ , For \quad C_y > 0 \\ f_2 = \dfrac{n_2}{T_2^{\frac{3}{2}}}\exp[\dfrac{-C^2}{T_2}]\ , For \quad C_y < 0 \end{cases} \quad (31)$$

These estimated distribution functions of the gas particles enable one to study their behavior in the investigated system that cannot be available by taking the way of the solution of Navier–Stokes equations. This will be the starting point to predict the irreversible thermodynamic behavior of the system in the next section.

The Thermodynamic Properties of The System

The problems of the thermodynamics of irreversible processes continue to arouse great interest (Ouaf, 2005 : Osalusi, 2007 : Shakhov, 1969 : Lebon et al., 2008 : Jou and Casas-Vázquez, 1993 : Shapirov 1995 : Sharipov, 1994). This is associated both with the general theoretical importance of this theory and its numerous applications in various branches of science.

Starting from the evaluation of the entropy per unit mass S , which is written as:

$$S=-\int f \; Log\, f d\underline{C} =\frac{\pi^{\frac{3}{2}}}{8}\left(n_1\left(3\text{-}4\ln\left(\frac{n_1}{T_1^{\frac{3}{2}}}\right)\right)+n_2\left(3\text{-}4\ln\left(\frac{n_2}{T_2^{\frac{3}{2}}}\right)\right)\right), \qquad (32)$$

The y-component of the entropy flux vector has the form

$$J_y=-\int C_y f \; Log f d\underline{C} =\frac{\pi}{2}\left(n_1\sqrt{T_1}\left[1-\ln\left(\frac{n_1}{T_1^{\frac{3}{2}}}\right)\right]-n_2\sqrt{T_2}\left[1-\ln\left(\frac{n_2}{T_2^{\frac{3}{2}}}\right)\right]\right)., \qquad (33)$$

while the Boltzmann's entropy production [(Ouaf, 2005 : Osalusi, 2007) in the steady state is expressed as :

$$\sigma_S = \underline{\nabla} \cdot \underline{J} \qquad (34)$$

Following the linear theory of irreversible thermodynamics, using Sharipove model (Shapirov 1995), we could estimate the thermodynamic force corresponding to the change in concentration:

$$X_1 = \frac{\Delta y}{n}\frac{\partial n}{\partial y}, \qquad (35)$$

the thermodynamic force corresponding to the change in temperature

$$X_2 = \frac{\Delta y}{T}\frac{\partial T}{\partial y}, \qquad (36)$$

where Δy is the thickness of the layer adjacent to the flat plate in units of the mean free path (the distance between two collisions of the gas particles)in the dimensionless form.

After calculating the thermodynamic forces and the entropy production we can get the kinetic coefficients L_{ij} from the relationship between the entropy production and the thermodynamic forces which has the form (Lebon et al., 2008):

$$\sigma_S = \sum_i\sum_j L_{ij} X_i X_j = \begin{pmatrix} X_1 & X_2 \end{pmatrix}\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}\end{pmatrix}\begin{pmatrix} X_1 \\ X_2\end{pmatrix}. \qquad (38)$$

The restrictions on the signs of phenomenological coefficients L_{ij} which arises as a consequence of the second law of the thermodynamics that yields the quadratic form (Lebon et al., 2008) :

$$\sigma_S = \sum_i\sum_j L_{ij} X_i X_j \geq 0, \qquad (39)$$

can be deduced from the standard results in algebra. The necessary and sufficient conditions for $\sigma_s \geq 0$ are fulfilled by the determinant

$$\left|L_{ij} + L_{ji}\right| \geq 0, \qquad (40)$$

and all its principal minors are non-negative too.

Another restriction on L_{ij} was established by Onsager (1931) is that, besides the restrictions on the signs, the phenomenological coefficients verify important symmetry properties. Invoking the principle of microscopic reversibility and using the theory of fluctuations, Onsager was able to demonstrate the symmetry property

$$L_{ij} = L_{ji}, \qquad (41)$$

which is called the Onsager's reciprocal relations (Jou and Casas-Vázquez, 1993 : Shapirov 1995 : Sharipov, 1994).

The Gibb's formula for the variation of the internal energy applied to the system (gas + heated plate) is

$$dU = dU_S + dU_V + dU_R \qquad (42)$$

where the internal energy change due to the variation of the extensive variables: entropy, volume and the temperature gradient produced by the thermal radiation field are respectively

$$dU_S = TdS, dU_V = -PdV, dU_R = \phi\frac{\partial T}{\partial y}\Delta y, \text{ where } \phi=\left(\frac{16\sigma T_\infty^3 T_s}{3cn_s K_B T_s}\right). \qquad (43)$$

The pressure and change in volume are

$$P = n\ T, dV = -\frac{dn}{n^2}, \text{and } \Delta y = 5 \text{ (say)}.$$

DISCUSSION AND CONCLUSIONS

In a frame co-moving with the gas, we have investigated the behavior of the gas under the influence of a thermal radiation field in the steady state of a plane heat transfer problem in the system (gas + heated plate). The thermal radiation is introduced in the force term in the Boltzmann equation in a linearized form, for the case of a neutral gas for the first time in the best of our knowledge. In all calculations and figures we take the following parameters values for the Argon gas:

$$\sigma_s = 5.6705\times10^{-5}\ erg/cm^2.\sec.K^4; Kn=1; R=8.3145\times10^7 erg/\text{deg-}mol;$$
$$\rho_s = mn_s = 6.633\times10^{-11} gm/cm^3; c = 2.9979\times10^{10} cm/\sec; n_s = 10^{12} cm^{-3};$$
$$\tau = 2.366\times10^{-3}\sec; T_s = 1000K, \frac{T_s - T_\infty}{T_s} = 0.04, V_{Th} = 6.4522\times10^4 cm/\sec;$$
$$\lambda = 152.64 cm; N = 0.00162,$$

we calculate all the sought variables in the radiation field intensities corresponding to the plate temperatures (T = 1000K). We will discuss the behavior of the gas particles in the non-equilibrium state. While the number density $n(y)$ increases with increasing the distance from the plate, the temperature $T(y)$ decreases. This is happen because when the application of heat to a gas causes it to expand , it is thereby rendered rarer than the neighboring parts of the gas ; and it tends to form an upward current of the heated gas , which is of course accompanied with a current of the more remote parts of the gas in the opposite direction. The gas is thus made to circulate;

fresh portions of gas are brought into the neighborhood of the source of heat, carrying their heat with them into other regions. In other words, heat will transfer from the hot surface into the gas, and the layer adjacent to the solid surface will be heated up. The next layer will also heated up, but to a lesser extent. In this manner, a temperature gradient will be set up across the gas. But the temperature gradient will create a density gradient in the reverse direction, that is, with the slightest gas density near the plate, see figures (1,2). Accordingly, the thermodynamic force due to the gradient of temperature X_T will have the opposite direction to the thermodynamic force due to the gradient of the density X_n , see figures (11,12) . A comparison between figures (3) and (4), shed light upon that the decrement f_2 and the increment f_1 of the distribution function f far from the equilibrium($\chi = 0.65$) compensate each other, from figure (5) it is compared with f_0 at equilibrium($\chi = 1$). This is interpreted as the departure from the equilibrium state, where the gas particles having temperature T_1 and density n_1 , after heated, is replaced by a counter change by the gas particles having temperature T_2 and density n_2 . This behavior agrees with the famous Le Chatelier principle.

It is shown from figures (6,7) that the entropy $S(y)$ is an increasing function and the entropy production $\sigma(y)$ is a nonnegative one for all values of y and χ . They satisfy the second law of thermodynamics. The behavior of the different contributions of the change in internal energies can be illustrated as follows; the internal energies changes caused by the variations in the temperature dU_S and radiation energy dU_R have a negative sign. This is due to that they have the same direction as the thermodynamic force X_T formed by the gradient of temperature and they decrease in magnitude towards the equilibrium state ($\chi = 1$), see figures (8,10,11).The internal energy change dU_V caused by the variation in density has a positive sign . This is because it takes the same direction as the thermodynamic force X_n due to the gradient of density. Also, as expected, it decreases towards the equilibrium state ($\chi = 1$), see figures (9,12). The numerical ratios between the different contributions of the internal energy changes based upon the total derivatives of the extensive parameters are predicted via the Gibbs's formula illustrated in figures (8, 9,10). Taking into consideration their different tendencies, the maximum numerical values of the three contributions at various radiation field intensity corresponding to various plate temperatures are ordered in magnitude as follows:

$$dU_s(1000K) : dU_V(1000K) : dU_R(1000K) \approx 10^2 : 1 : 1.$$

According to our calculations, the restrictions imposed on the kinetic coefficients L_{ij} are satisfied where $L_{11} \geq 0$,

$L_{22} \geq 0$ and $L_{33} \geq 0$.The celebrated Onsager's reciprocal relations are satisfied, where $\left(L_{12} \equiv L_{21}, L_{13} \equiv L_{31} \text{ and } L_{32} \equiv L_{23}\right)$. By using Sharipove's model [49] we find that the Onsager's inequality is fluctuating in the order of $\pm 10^{-14}$ which is a very acceptable error.

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NOMENCLATUR

C	The velocity of the gas particles.	T_s	The temperature at the plate surface.
F_y	The thermal radiation force component along y-axis direction.	T_∞	Temperature far from the plate surface.
J_y	The entropy flux component along y-axis direction.	T_1	The temperature related to f_1.
K_B	Boltzmann constant.	T_2	The temperature related to f_2.
K	Hydrodynamic Kundsen number	u	The flow(bulk) velocity
		U	The internal energy of the gas.

V	The gas volume.	n_s	The concentration at the plate surface.
V_{Th}	Gas thermal velocity	n_1	The concentration related to f_1.
X_1	Thermodynamic force corresponding to variation of concentration.	n_2	The concentration related to f_2.
X_2	Thermodynamic force corresponding to variation of temperature.	y	displacement variable.
c	The velocity of the light.	**Greek symbols**	
f	The distribution function.	λ	The mean free path
		τ	The relaxation time.
		μ_s	The gas viscosity .