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Full Length Research Paper

Knowledge level of undergraduate students of mathematics teaching on proof methods

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The study aimed to determine knowledge levels of undergraduate students of mathematics teaching on proof methods. The study was carried out on a total of 199 students, 24 of which were females and 75 of which were male, studying at the department of Mathematics Teaching at Kazım Karabekir Faculty of Education in Ataturk University. A concept testing consisting of seven open-ended questions on proof methods widely used in courses which aimed to analyze knowledge of students on proof methods was used for data collection.

Keywords: Undergraduate students, mathematics teaching, proof methods.

INTRODUCTION

In mathematics, axiom is a proposition that is considered to be true without proven or demonstrated truth, while theorem is a statement that has been proven. Theorems and proofs of theorems are one of the most important components of mathematics. It is mentioned in school programs that proof plays and important role in mathematics (Varghese, 2011). In addition, the researchers concentrate on the latest innovations in mathematics teaching and the importance of proof and reasoning in mathematics teaching (Varghese, 2009). Theorems and their proofs can be regarded as very important tools which take one to the roots of mathematics and show the solution of all unknown problems. Bretscher emphasized the value of theorems and proofs by saying "mathematics without proof make the subject loses its essence." (Bretscher, 2003).

While the knots in mathematics are solved, theorem is proven by various proof methods. The aim of all proofs is to prove if what is claimed is true or false. Mathematical proves are generally made in three stages. In stage one, whether the statement is true is investigated; in stage two, why the statement is true is explained and the final stage, abstraction is made by suggesting generalization conditions (Baki, 2008)

In interviews on school mathematics, previous researchers emphasized the importance of reasoning and proof for the construct of mathematical understanding (Bieda,2010) It should not be surprising to concentrate on the importance of proof, which is a must for mathematics. Students acquire benefits from the proof of theorems. Varghese suggested that since mathematical learning required detection, proof and reasoning were among the strong methods to improve comprehension, correlation and development of mathematical communication (Varghese, 2009). Thinking processes defined by NCTM (2000) National Council of Teachers of Mathematics include the concept of proof in addition to the concepts of problem solving, relation, representations, communication and reasoning (Suh, 2010).

Concepts and the definitions of concepts are often used during theorem proving. Thus, the concept, which is an important construct in mathematics teaching and the definition of concept and the relationships between the concepts can be reinforced

(Bingolbaliand Monaghan;2008). Mathematics becomes attractive since the problems in theorems would be discussed (Freiman, Manuel and Lirette; 2007). As theorem proving forms as basis to discuss mathematical opinions of the students, the teachers has the opportunity to improve mathematical opinion of the students (Barlow and Mccory; 2011). At each step of theorem proving, beliefs, attitudes and emotions which are important for mathematical learning and teaching unravel on condition that these positive cases are achieved and motivation for further learning can be encouraged (Caballero, Blanco and Guerreo; 2011).

Considering the benefits of theorem and proof, it becomes compulsory to introduce the students with theorem and proof in mathematics courses throughout their learning period. For this reason, in parallel to the readiness of students, the concept of proof is provided in every step of education. Although the concept of proof is included in mathematics courses in various grades of education, they particularly serve as a basis in university mathematics courses (Mejia-Ramos, Fuller Rhodas and Samkoff; 2012). In an environment where the components of teachers' knowledge, pedagogic content knowledge, general pedagogic knowledge are discussed (Koniq, Blomeke, Paine, Schmidt and Hsieh, 2011), like in other branches, this is valid for the education of mathematics teachers or mathematicians. If a person who will be specialized in the field of mathematics knows all information including theorem and proofs in a very detailed manner, he/she will have a rich content knowledge.

In this case, one of the things to question is the extent of mental organization or mental ability of a student related to proof methods of any theorem. It involves theorem proving methods and the skill of knowing when and how to use these methods.

METHOD

Sampling

The sampling of the study consisted of a total of 199 students, 24 of whom were female and 75 of whom were male. The students were studying at the Department of Mathematics Teaching at Atatürk University, Kazım Karabekir Faculty of Education.

Data Collection Tool

A concept testing consisting of seven open-ended questions involving proof methods widely used in courses which aimed to analyze knowledge levels of students on proof methods was used for data collection.

Data analysis

The answers of the students to open-ended questions were evaluated as "correct", "partially correct" and "incorrect". Chi-square analysis was used for statistical analysis.

Findings and comments

Findings and comments to the answers to the questions in concept testing used in the study are presented below.

12.9% of female students gave correct answers and 87.1% gave incorrect answers; while 13.3% of male students gave correct answers and 86.7% gave incorrect answers to the question "What is theorem proving? Explain." The answer "partially correct" was not found among the answers of the students. The implication $p \Longrightarrow q$ with true p hypothesis is called a theorem. Theorem proving means reaching the statement based on the hypothesis of the theorem. It was found that 13.1% of the students had this knowledge, while the rest did not have any knowledge on this subject even partially. The answer "$p \Rightarrow q$" implication is called hypothesis" was among incorrect answers.

The answers to the question "Give information about induction method" are presented in table 2. It is understood from Table 2 that, 47.6% of female students gave correct answers; 5.6% gave partially correct answers and 46.5% gave incorrect answers to the question. On the other hand, it was found that 58.7% of male students gave correct answers; 4% gave partially correct answers and 37.3% gave incorrect answers to this question. Proof method in the form of reaching the general judgment from special cases is called induction method. This method can be explained as follows:

P(n) is a proposition about natural numbers and D is the set of truth values of this proposition, in other words;

Let D={n: n ∈N and P(n) } be correct, If,

a)

∈D

b)

∈D then if (k+1) ∈D then D=N. Statements such as "we should show that (k+1) ∈D considering that 1∈D" were observed among the answers of the students.

Table 1

Gender		Correct	Incorrect	Total
Female	N	16	108	124
	%	12.9	87.1	100
Male	N	10	65	75
	%	13,3	86.7	100
Total	N	26	173	199
	%	13.1	86.9	100

D.F. = 1 χ^2 = 0.008 p>0.05

Table 2

Gender	N %	Correct	Partially Correct	Incorrect	Total
Female	N	59	7	58	124
	%	47.6	5.6	46.8	100.0
Male	N	44	3	28	75
	%	58.7	4.0	37.3	100.0
Total	N	103	10	86	199
	%	51.8	5.0	43.2	100

D.F. = 2 χ^2 = 2.325 p>0.05

Table3

Gender		Correct	Incorrect	Total
Female	N	25	99	124
	%	20.2	79.8	100.0
Male	N	11	64	75
	%	14,7	85.3	100.0
Total	N	36	163	199
	%	18.1	81.9	100

D.F. = 1 χ^2 = 0.952 p>0.05

Table 4

Gender		Correct	Incorrect	Total
Female	N	36	88	124
	%	29.0	71.0	100.0
Male	N	27	48	75
	%	36,0	64.0	100.0
Total	N	63	163	199
	%	31.7	68.3	100

D.F. = 1 χ^2 = 1.049 p>0.05

Among partially correct answers, the first two steps of induction methods was written correctly or the first step was written correctly but the third and final step (k+1) $\in$D was written incorrectly.

As for the question “What is direct proof? Explain.” It was found that 20.2% of female students gave correct answers; 79.8% gave incorrect answers. On the other hand, 14.7% of male students gave correct answers,

Table 5

Gender		Correct	Incorrect	Total
Female	N	5	119	124
	%	4.0	96.0	100.0
Male	N	3	72	75
	%	4.0	96.0	100.0
Total	N	8	191	199
	%	4.0	96.0	100

D.F. = 1 χ^2 = 0,000 $p>0.05$

Table 6

Gender		Correct	Incorrect	Total
Female	N	41	83	124
	%	33.1	66.9	100.0
Male	N	24	51	75
	%	32.0	68.0	100.0
Total	N	65	134	199
	%	32.7	67.3	100

D.F. = 1 χ^2 = 0.024 $p>0.05$

Table 7

Gender		Correct	Incorrect	Total
Female	N	74	50	124
	%	59.7	40.3	100.0
Male	N	55	20	75
	%	73.3	26.7	100.0
Total	N	129	170	199
	%	64.8	35.2	100

D.F. = 1 χ^2 = 3.822 $p>0.05$

while 85.3% gave incorrect answers. The answer "partially correct" was not used by female or male students. In calculation of theorem $p \Rightarrow q$ using direct proof method, the truth of propositions such as $p \Rightarrow p_1$, $p_1 \Rightarrow p_2$, $p_2 \Rightarrow p_3$, $p_n \Rightarrow q$ are showed and it is implied that the proposition $p \Rightarrow q$ is true. Incorrect answers included defining direct proof as the method to obtain the proposition $p \Rightarrow q$, using the propositions $p \Rightarrow q^{/}$, $q^{/} \Rightarrow r$, $r \Rightarrow q$. In addition, although they are totally unrelated, some of the students confused proof by contradiction, induction, trial and error and counter example methods with direct proof method.

As for the answers given to the question "Explain proof by transposition method", it was found that 29% of female students gave correct answers; 71% gave incorrect answers, while 36% of male students gave correct answers and 64% gave incorrect answers. The answer "partially correct" was not used by the students. In this proof method the equation $p \Rightarrow q$,$q^{/} \Rightarrow p^{/}$ is used. The method of proving a theorem with contrary reverse instead of direct proof is called proof by transposition method. It was observed that the students wrote proof by contradiction method as reductio ad absurdum method in incorrect answers. Some answers defined direct proof method instead of reductio ad absurdum.

Data related to the question "Explain proof by contradiction method" is presented in Table 5. 4% of female students gave correct answers, while 96% gave incorrect answers. On the other hand, 4% of male students gave correct answers, while 96% gave incorrect answers to the question. The answer partially correct was not used by the students. Proof by contradiction is a form of proof that establishes e general contradiction based on a proposition's being false in a theorem. In this proof method, the equation $(p \Rightarrow q)^{/}$, $p \wedge q^{/}$ is used. The principle is to obtain contradiction from the falsity of the statement in proof by contradiction method. The majority of the students, (96%) gave incorrect answers to this question. Analysis of incorrect answers showed that generally it was claimed that $p \Rightarrow q$ is equal to $p/\wedge q$, $p/\vee q/$. In addition, it was found that definitions of other proof methods were confused with proof by contradiction method.

Findings about the question “Explain trial and error method” are presented in Table 6. According to the table, 33.1% of female students gave correct answers, while 66.9% gave incorrect answers. On the other hand, 32% of males students gave correct answers; and 68% gave incorrect answers. The answer partially correct was not used by the students. Proof method with trial and error method can be used in a proposition which takes different values. These values are written in their different places and the truth of the proposition is checked. One of the students defined this method as “the method applied by using each of the different proof methods of theorem” while another student defined this method as “trial and error method is one of the proof methods which accepts the truth or falsity of for example our hypothesis”.

As for the answers to the questions “Explain counter example proof method”, the following findings were obtained. Of the female students, 59.7% gave correct answers, while 40.3% gave incorrect answers to the question. As for males, 73.3% gave correct answers, while 26.7% gave incorrect answers to the question. The answer partially correct was not used by the students. In counter example proof method, by finding minimum one value showing that the given proposition was not true, the falsity of this proposition is proven. This method is generally used to prove that a proposition such as $p \Rightarrow q$ is false. Incorrect answers showed that counter example proof was confused with proof by transposition and proof by contradiction methods. In addition, a statement such as “ the proof is completed by giving one example showing that falsity of another proposition equal to the given proposition is false” was among the answers.

In addition to above mentioned findings, chi-square values for the differences between the answers of female and male students to all questions were found to be statistically insignificant.

RESULT AND SUGGESTIONS

If the students know the implication “$p \Rightarrow q$” as hypothesis, we should firstly make the students grasp the concept of theorem. We should make them recognize the concepts of hypothesis and statement. In other words, we should describe and explain qualities of theorem in full. After this stage, we should teach the meaning of theorem proving in general terms and we should make them know about the stages used in theorem proving. In conclusion, a general information should be given about theorem and theorem proving. We can only contribute to cognitive development in this manner. Cognitive development about a concept is related to effective development (Panaoura, Gagatsis, Deliyianni and Elia, 2009).

It was observed that the students wrote irrelevant things instead of the proof methods which were asked to be described. This finding evokes the opinion that majority of the students do not have an idea as to proof methods, where and when they are used. To eliminate this, particularly the students at junior year in mathematics teaching should be extensively informed about theorem proof methods and then courses involving theorems and proofs should be taught.

It was observed that 96% of the students gave incorrect answers to the question “Explain proof by contradiction method.” Analysis of incorrect answers showed that the students generally claimed that $p \Rightarrow q$ was equal to $p/\wedge q$, $p/\vee q/$. It was found that the students had no knowledge about $p \Rightarrow q$ conditional proposition and as a result, they failed to give desired answers. In addition, since proof by contradiction method required more information than other proof methods, it was observed that success ratio was rather low. In this case, the subject of logic and all sub-concepts related to the subject should be adequately taught to the students.

Majority of undergraduate students of mathematics teaching have difficulty in writing mathematical proof (Powers, Craviotto and Grassl, 2010). Students generally do not know how to start proof (Bell2011). In an attempt to help the students to overcome this difficulty, the teacher should start by choosing correct course books. Course books play an important role in students’ comprehension of reasoning and proof (Jhonson, Thampson and Senk; 2010).

The fact that the answer “partially correct” was given in induction proof method and relatively higher number of correct answers in this proof method can be explained as follows: when we, the teachers, are going to explain a theorem with the methods apart from induction, we apply this method directly without explaining to the students. When we are going to prove the theorem using induction method, on the other hand, we say “Let’s now prove this with induction.” As a result, induction method might be more familiar than other proof methods. For this reason, it is believed that it would be beneficial if the teachers explain which proof method they will use for theorem proving and if they make a short description of the method before they start theorem proving.

During the course, the teachers should explain which variables they will take into account during theorem proving and why they would use those variables; then they should continuously question the students about these concepts for the proving of next theorems. Thus, mathematical discussion, which is an important skill shaping proof processes that students should acquire will also develop (Hoffman, Breyfogle and Dressler 2009). Since contemporary educational approaches expect the students to actively produce knowledge (Psycharis, 2011), it will be actively produced by students.

The question one should ask is: How can a student be successful despite this lack of knowledge about theorem and proofs? This can be explained as follows: study methods of the students are based on a reasoning based on the copying of proof (Baştürk, 2010). In other words, it is based on memorization in a sense. The students try to succeed in theorem proving by memorizing instead of comprehending it. To prevent this, in examinations, instead of writing a theorem and asking the students to explain it, questions explaining what is asked from the theorem, how a theorem should be proven and open the closed sections should be asked. Thus, the students can understand that they should comprehend the theorems instead of memorizing them.

Although the students had serious difficulty in theorem proving, they had limited information about how to overcome these difficulties (Staylianides and Stylianides; 2009). Intensive and various studies should be carried out in this subject and the students and teachers should be informed.

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