Full Length Research Paper

Multi-criteria and Forecasting Tools in a Case Study under the Participatory Planning Guidelines

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This article presents the conclusions from applying a procedure based on multi-criteria and qualitative forecasting methods to hierarchising transport infrastructure investment options with real data; it focusses on the key steps. The procedure can be generalised to urban and regional planning and so on. One important aspect is that it requires multi-segment participation on the basis of participatory planning guidelines, which gives the stakeholders involved greater confidence in the results and in the likelihood they will actually be implemented. The article discusses salient aspects of related theory. A pilot case study was conducted in Brazil as a preliminary to a broader study (later conducted) to validate the procedure and prepare the way for generalisation to other decision-making situations. It was observed that options that obey eco-sustainability and social criteria were preferred, about 24% of the results favouring investments in railways, whether for cargo or passenger transport.

Keywords: Transportation infrastructure planning; Multi-criteria methods; TOPSIS; AHP; DELPHI.

INTRODUCTION

This paper revisits an important subject: how to decide among alternative investment projects when there are funding constraints on implementation within a single planning timeframe. Accordingly, the projects would carried out on the basis of cascaded planning, in that the most significant, urgent and complementary gain priority for execution. In order to assure agreement in the final results presented to the decision-makers, guidelines for participatory planning by the main stakeholders involved in the problem addressed were observed in choosing a solution procedure. Therefore, some advantages of the participatory planning directives that guided how the study was conducted are then cited. These influenced decisions on the solution procedure adopted for the specific study undertaken to hierarchise transport infrastructure projects necessary in Brazil, and thus define an execution programme. In the brief literature review presented here it was possible to recognise classical methods applied in the past that could be chosen to make up the procedure and thus inform verification of the resulting procedure’s applicability to analysing investment planning decision making. Other more recent applications of the chosen methods are also cited, mainly because they reinforce the appropriateness of the choice made for the focused case study for planning infrastructure investments in Brazil.
Participatory planning has been employed in establishing priorities and choices in various fields of management. Several methods can be used in conducting that activity. Chambers (2005) states that people and authorities should be encouraged to conduct participatory planning, in that way accepting responsibilities and commitments and sharing ideas. Therefore, it is increasingly being used in various sectors of the economy, and numerous information and communication technology (ICT) resources are becoming available that make this behaviour increasingly feasible and less costly. In the government planning sector, for example, choosing among public and private investment alternatives should result in participatory planning involving stakeholder decision-making and operational levels, so as to improve the credibility of, and reduce resistance to, outcomes obtained by the co-ordination of a technical team.

As should be clear, the recommendation of participatory planning is by no means restricted to government problems: the main task continues to be to choose what – preferably simple, but effective – technique or method (or even a combination of them) to select, because in fact there are often several levels of decision-making, and the final decision is taken after analysing outcomes from a decision-making process and not simply from the conclusions obtained through some solution methods.

For years, researchers in decision-making have faced the challenge of turning qualitative externalities into quantitative data (Silva 2008; Silva & Cavalcanti Netto 2012). This concern has produced creative methods classified as Multi-Criteria Decision Making (MCDM) (Fishburn & Lavalle 1999). This review starts with Goal Programming (Charnes & Cooper, 1961), based on Koopmans (1951), with regard to the efficiency vector used in Multi-Objective Mathematical Programming. This was followed by ELECTRE I (Benayoun et al. 1966) in five successive versions. Lee (1972), Ignizio (1976) and Keeney & Raiffa (1976) presented new proposals; Saaty (1980) produced the Analytic Hierarchy Process (AHP), based on pair-by-pair evaluation of criteria and application of the evaluated options through interaction with stakeholders. Lootsma (1993) proposed the Multiplicative AHP, a variation in which the transposition of a verbal scale to numerical values (used in the AHP) was replaced by numerical values on a geometrical progression with a progression factor of 2. Hwang & Yoon (1981) presented the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), in which m alternatives are evaluated by n priority-assessment attributes determined by an indicator derived from the combination between closeness to an ideal (positive) situation and distance from a non-ideal (negative) situation. This method was enhanced by Yoon (1987) and Hwang & Liu (1993). PROMETHEE I (Brans 1982) was developed through versions I, II, III, IV etc. Behzadian et al. (2010) presented a review of PROMETHEE methodologies and applications. VIKOR (Opricovic & Tzeng 2007) is similar to TOPSIS; both evaluate alternatives in terms of distance between ideal and non-ideal values. One difference between the two is the normalisation of the main matrices: TOPSIS employs vector normalisation; VIKOR, linear normalisation. One vulnerability of TOPSIS is the potential for error present in the criteria weightings. In attempts to neutralise this, Saghatian & Hejazi (2005), for example, used triangular fuzzy numbers to minimise distortions in attributing values to criteria.

Other studies reviewed include Janic & Reggiani (2002), evaluating hub airports; Soo et al. (2006), regarding traffic signal control investments; and Liang (2007), applying fuzzy logic to a production/transportation problem.

AHP and TOPSIS have been applied jointly. Even without extending the timeframe of this review, studies using fuzzy logic were seen, some with AHP, others with TOPSIS. Tzeng et al. (2005) evaluated alternative fuels for buses in public transport, comparing a high number of alternatives with other applications: the results of using the DELPHI and AHP methods were treated statistically to obtain mean values, and, finally, TOPSIS was applied on the basis of values obtained by experts. The VIKOR method was also used to validate the results.

Önüt & Soner (2007) conducted a study to choose appropriate sites for solid waste disposal, using AHP to determine weightings to apply to the TOPSIS matrix of initial evaluations defined by triangular fuzzy numbers. Isiklar & Büyüközkân (2007) used both approaches to choose alternatives among mobile telephones. Gumus (2008) used AHP and TOPSIS to select hazardous cargo transport firms, drawing on experts and using the DELPHI method (Sullivan & Claycombe 1977). İç & Yurdakul (2008) applied the methods jointly to priority-setting in choosing machining centers. Lin et al. (2008) conducted a theoretical study of palmtop design to suit consumer needs. Wu et al. (2008) presented an application of AHP with TOPSIS for priority-setting in choosing the best alternative insurance sold by banks. Other studies – including Tsaur et al. (2002), on evaluating airlines; Chen & Tzeng (2004), on evaluating countries for conducting international business; Yang et al. (2007), for allocating personnel to production lines; and Sheu (2008), on managing global supply chains – show that it is appropriate to combine AHP and TOPSIS when subjective considerations need to be transformed into numerical references for consistent evaluation.

Some recent studies reinforce the case for using TOPSIS and AHP in combination to hierarchise options in various different areas. These include Li et al. (2011), Collan et al. (2013), Sorayaei et al. (2013), as well as Ravi (2011) and Gao & Hailu (2013). Vinay et al. (2012) developed a method where AHP is used to prioritise...
goals, the output being an input to the TOPSIS. To conclude this brief review of more recent applied work, Nooramim et al. (2012) uses and compares results of AHP and TOPSIS techniques to choose the most efficient operating system in terminals.

This paper is organised as follows: the main stages of the procedure are described, and then validation is presented. Together with the key stages, theory is discussed in the light of controversies in the literature on the methods, which nonetheless have been no impediment to their widespread use in applied research. Some maths and features of the stages of the chosen methods are presented in the Appendix, by way of additional clarification. Interesting results of the prior pilot study of a real case for planning transportation infrastructure investments in Brazil are presented (Silva 2008; Silva & Cavalcanti Netto 2012; Silva et al. 2014). The case study was conducted with support and a grant from the Brazilian government transportation authorities, given the major importance of the subject to city planning and regional planning for Brazil’s present and future sustainable development. That pilot study validated the multi-criteria approach tool and the qualitative forecasting method in a real situation, which included and emphasised ethical and ecological issues, from the main premises of the first stage in the procedure, when the board of decision makers are the main work-team. In fact, in the study with real data and the participants actually involved, the eco-sustainable transportation investment projects are prioritised, as well as the corresponding selection criteria. These criteria could also be obtained by Multivariate Statistics, or more specifically Factor Analysis, which guarantees that a large number m of attributes/criteria can be aggregated on a small number n of uncorrelated attributes, depending on the analysis of the correlation matrix among these m attributes. This can be seen in the applied work of Murphy et al. (1992), as well as in Kline (1997) and Mingoti (2013). Lastly, after presenting intermediate and final outcomes of the procedures, recommendations are made for future applications and generalisation of the decision-analysis support tool. The choices of methods are justified by the context and characteristics of the problem.

PROCEDURE

Based on a literature review, a procedure was proposed comprising seven main stages (Figure 1) and a mix of methods in sequence, deploying the simplicity of some and the precision afforded by another. Accordingly, the chosen methods are grounded in prescriptive models also used in diverse experiments and reported in highly credible journals, for application in the transportation planning environment.

The key overall characteristics of the procedure are:

- participatory planning;
- a high level of stakeholder interaction with the problem environment and principles or premises, some ethical;
- simple use of the methods; and
- methods understood in spirit by decision-makers, not only by the group of analysts, as well as for other applications and not only in the test study and its well-developed theoretical construction; and
- availability of off-the-shelf computational packages.

Thus, starting from the primary investigation, the study goes on to extract knowledge from various sources, experts and authorities on the basis of the various stakeholders’ expertise, to bring expectations and vision into convergence. The proposed procedure outputs a listing of a set of projects with highest priority for execution over time. Their impacts and/or influences are reported by evaluation on the basis of project criteria, both those obtained initially and those obtained in the final step of the same procedure, with their respective weightings.

Thus, after a ‘brainstorming’ stage, in which high-level decision-making participants start the process of selecting alternatives and necessary projects, defining priority-setting rules as main guides and assumptions, classic DELPHI (Appendix C) is applied to reduce the list of choices quite precisely, refine criteria and define their initial weightings for overall evaluation of investments. This is followed by the simple, objective TOPSIS ranking method (Hwang and Yoon 1981), in order to obtain a first hierarchical list of projects for execution.

Given the limitations and characteristics of the combined methods applied in the proposed procedure, after applying TOPSIS (Appendix A), some non-conflicting characteristics of the original method of Saaty (1980) (Appendix B) are used in greater detail, weighting the criteria by means of pair-by-pair evaluation of criteria by project (Netto et al. 2014).

Bana e Costa & Vansnick (2001), Saaty (2010) and Gomes (2003) offer arguments for and against the appropriateness of the use of eigenvalues and eigenvectors in Saaty’s original method. For example, the former two authors claim that their paper addresses ‘a fundamental problem in using the principal eigenvector of a positive pairwise comparison matrix to derive priorities’ – and, consequently, the eigenvalue – and that this ‘violation’ is characterised in their study. This is refuted by Saaty (2010), in an explanation in layman’s terms, with examples, stating that ‘in the Analytic Hierarchy Process (AHP) for decision making the criteria and alternatives are prioritised by forming matrices of judgments and from these judgments priorities are derived for each matrix in the form of the principal eigenvector. An eigenvector is a technical mathematical idea that would benefit from a simplifying explanation’, as is described in his study,
**1-Stage 1: Actions of State**
Assumptions and initial project list (Executive and other powers: interviews, debates, forums, brainstorming...with a technical team interacting and conducting the interviews).

**2-Stage 2: Formalisation of DELPHI method**
Selection of respondents among agents of change, agency representatives and social stakeholders, with full independence and no knowledge of each other; and formalisation of questionnaires based on initial assumptions and initial project list to redefine this initial list and define main criteria and their weightings (only non-conflicting criteria are considered) to be used to rank the projects for execution (with support from the technical team).

**3-Stage 3: Classic TOPSIS application**
Application of the TOPSIS ranking method to the projects, using the weightings from stage 2.

**4-Stage 4: Defining project weights**
Partial application of AHP to precise project weightings, after pruning the list of projects ranked in stage 3.

**5-Stage 5: Defining more accurate project weights by project type and by region**
From partial AHP results, regrouping weightings by project type and by project regional importance.

**6-Stage 6: Applying TOPSIS with more precise weights**
TOPSIS + AHP, which means using partial AHP results (i.e. the weightings from stage 5 as the weights input to classical TOPSIS) and a full list of projects and comparison of results of stage 3 and 6 (project ranking).

**9-Stage 7: Other support for final decision-making**
Final report presented to decision-makers: results from the procedure, mappings, other methods for each project selected which make it possible to revise its position in the project execution priority ranking – such as the Cross impacts method (Silva, 2008) – final proposals and remarks from the analysts or the technical team.

Source: adapted from Silva & Cavalcanti Netto (2012).

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which reinforces its use or appropriate use. Meanwhile, Montevechi & Pamplona (2013) examine the steps of the Saaty method, the use of eigenvalue and eigenvector, and undertake a simplified application. Also, as established in Saaty (2010, p. 163), the decision-making process is based on comparison judgments expressed in terms of declared preferences between pairs of options. If there are many criteria and many alternatives, this entails a need for synthesis obtained from a supermatrix, and thus the largest eigenvalue is obtained, whose corresponding principal eigenvector is the synthesis of all other eigenvectors.

The controversy over whether it is reasonable to establish a consistency ratio in AHP made it uncomfortable to continue its generalised use in this present study without reassessing what it means to calculate it and what its significance is in analysing the consistency of pair-wise judgements of importance between criteria, by option or by project, evaluated in the starting square matrix and the normalised matrix. In this respect, Wanderley (2014), drawing on Saaty (1977) and Montevechi & Pamplona (2013), re-established the mathematical line of thinking of the author of the AHP method. Accordingly, the essentials of the mathematical analysis of the real value of the consistency ratio are described briefly below, together with the use of the eigenvalue as the basis for calculating it:

Let there be a positive reciprocal matrix $A$ of order $N$, where the $a_{i,j}$ elements are given by Eq. (1), where $w_i$ is a weight vector.

$$a_{i,j} = \frac{w_i}{w_j}$$

It can be proven by simple matrix multiplication that:

$$A\vec{w} = N\vec{w}$$

or

$$(A - N\text{I})\vec{w} = 0$$

Therefore, $N$ is the maximum eigenvalue of the matrix $A$, and all the other eigenvalues are null.

In the AHP method, however, the vector of the $w_i$ is not known. In fact, we have an estimate of the comparison matrix $A$ and wish to obtain the intangible benefits vector $\vec{w}$. As we have only an estimate of the comparison matrix, the result obtained in Eq. (3) takes the form of an eigenvalue and eigenvector problem, as in Eq. (4), where the eigenvector is an estimate of the intangible benefits and the maximum eigenvalue is a measure for obtaining the consistency of the judgements in determining the matrix $A$. Now, $\vec{w}'$ is the eigenvector of the estimated comparison matrix (Wanderley, 2014):

$$(A - \lambda_{\text{max}} I)\vec{w}' = 0$$

Saaty (1977) uses the consistency ratio $RC$, shown in the following equation to determine the consistency of the
judgements in determining the matrix $A$, where the consistency index $IC$ is also defined in terms of the maximum eigenvalue. In Eq. (5), $CR$ is a random consistency index obtained and presented by Saaty (1991).

\[
RC = \frac{IC}{CR}
\]

\[
IC = \frac{\lambda_{\text{max}} - N}{N - 1}
\]

Therefore, if the calculations produce $RC$ less than 0.10, it is recommended that the judgement matrix $A$ be reviewed by option evaluated or by project.

**Description of the procedure**

**First stage**

At this stage (actions of State), policy decisions are made regarding Brazil’s transportation infrastructure needs and environmental constraints. Government departments and agencies – pressured by public opinion, business, the Legislative, trade unions and professional associations, environmental agencies and other stakeholders – set guidelines (premises to be obeyed) for developing a strategic plan, listing the main project investments and preparing an initial list of $m$ projects.

**Second stage**

At this stage, the technical team plans and executes actions to extract knowledge from stakeholders/experts working within the government’s guidelines and project investment preconditions.

They carefully interact with the chosen group of experts, using appropriate questionnaires and without interconnection among the experts (see Silva, 2008). As a result, they define the composition of the set of service attributes associated with the projects execution or criteria, and their weightings, for ordering and selecting the projects and priorities to be associated with them. The technical team uses the results from DELPHI also to adjust the actual list of projects. At this stage, criteria defined in view of the government strategy preconditions are based on their experience and expertise, taking into consideration the weightings allocated to the preconditions and the initial list of projects.

Sullivan & Claycombe (1977) pointed out that, despite the many promising features of the DELPHI method, it has many undesirable features, such as the items listed below by Sackman (1974):

[...]

1. often characterised by crude questionnaire design;
2. lacking in minimal professional standards for opinion item analyses and pilot testing;
3. highly vulnerable on its concept of ‘expert’ with unaccountable sampling and vulnerable in the selection of panelists, expert or otherwise;
4. virtually oblivious to reliability measurement and scientific validation of findings;
5. typically generating snap answers to ambiguous questions representing inkbLOTS of the future;
6. seriously confusing aggregations of raw opinion with systematic prediction;
7. capitalising on forced consensus based on group suggestion;
8. giving an exaggerated illusion of precision, and misleading uniformed users of results;
9. indifferent to and unaware of unrelated techniques, psychometrics, group problem solving, and experimental design;
10. denigrating group and face-to-face discussion, and claiming superiority of anonymous group opinion over competing approaches without supporting proof.

Application of DELPHI is relatively low-cost, and this is one of the advantages of using it. Another advantage is the possibility of using Internet resources such as e-mails, social networks, data clouds and so on. A flowchart of the DELPHI procedures is shown in Appendix C. There is also a general consensus that the DELPHI method can be a useful tool to preview future events, but its limitations must be taken into account; for example, the difficulty in correlating future variables explicitly (Sullivan & Claycombe 1977).

**Third stage:**

At this stage, TOPSIS (Hwang and Yoon 1981) (Appendix A) entails executing the method’s algorithm and the assignment of priorities to the alternatives (projects) by using a single table of weightings for the chosen criteria obtained in the DELPHI stage. The chosen respondents are the same as in the anterior stage.

At this point, after careful analysis, researchers pre-define a plan to prune the list of projects already prioritised. The goal is to reduce the number of projects to be evaluated in the next step, generally to a number $n-m$ of options. The size and type of problems that require application of this procedure to the ranked list of projects for execution are particularly important given the practical limitations of the next stage. For instance, when outlining the plan for pruning the list, all projects types in each region of the country must be represented. In the case of Brazil, considering its continental reach, evaluation needs vary greatly. In any case, the output generated from this stage is the first list of projects, ranked from highest to lowest
Fourth stage

At this stage, application of partial AHP defines a priority matrix or weighting matrix, and projects are evaluated in terms of the relative importance of one criterion over another, based on the desired assumption and without correlation with the set of criteria chosen (Saaty 1980; 1991), on the basis of the numerical scale of the table in Saaty (1991; 2011). At this time, a random index is also established from simulations performed previously (Appendix B), where characteristics of the Saaty method and its contribution over time are further detailed. When the degree of importance of one criterion over another is defined, with degrees ranging from 1 (equal importance) to 9 (absolute greater importance), with intermediate values of 2, 4, 6 and 8, the index is automatically established and the consistency ratio obtained. Thus, an appropriate evaluation with AHP provides each project with a specific criteria weighting table.

In this regard, Vargas (1990) advises that, to conduct such AHP evaluations, it is necessary to assure methodological consistency by observing the following axioms:

**Reciprocal comparison**

Decision makers should be able to make comparisons and establish their preferences clearly. These preferences must be subject to the condition of reciprocity. Thus, if A is $n$ times preferable to B, that means B is $1/n$ times preferable to A. If these conditions are not met, there is an indication that the issue raised is not clear and should be reconsidered.

**Uniformity**

Data to be compared should be on a limited scale. If they are not a homogenous group, they must be grouped into elements with the same order of magnitude. If the decision maker cannot provide a satisfactory answer, the question is not significant or the alternatives offered are not comparable due to lack of homogeneity.

**Independence**

After preferences are stated, it should be admitted that the criteria used are independent of the properties of the alternatives, which should not, therefore, influence the decision maker.

**Expectation**

It is essential that the hierarchical structure be complete so that decision makers are able to express their preferences, otherwise they cannot employ the necessary criteria to meet their rational expectations.

Fifth stage

This stage, which can be called mean AHP, considers the results of the previous stage, associating the individual weightings with transportation project type (road, rail, water, air etc.) by area or country region (project groups). The arithmetic means of the project group weightings are calculated, and the standard deviations allow validation of results. The output of this stage is a table of criteria weightings for project type categories by region or mode type groups (road, rail, water, bridge or viaduct, rail or highway bypass, for instance) or the mean weightings.

Sixth stage

This involves a combination of AHP and TOPSIS. TOPSIS is reapplied with the weightings by group of projects obtained in the fifth stage. Thus, taking the initial list of alternatives, where complementarity between projects may or may not be observed, the entire list of projects is reordered by priority. This outcome can be compared with the first outcome: the ordered list that used the first table of weights by criteria (from the DELPHI stage) for the classical TOPSIS application. Thus, this step of the procedure generates the second output of this stage: comparing the orderings, which is one of the final outcomes to be evaluated in the next stage in order to generate a final report of execution priorities for implementation.

Seventh stage

This stage comprises evaluation and proposals. The outputs are analysed, comparing the classic TOPSIS with the combination of TOPSIS and AHP of the sixth stage. Notice that even considering this as one outcome, it is not the really desired target i.e. the final list ranking execution by place and time. The project rankings obtained with the individual and mean weightings are then compared, and the results can be validated by other qualitative methods such as the Cross-Impact method (Schlange & Jüttner 1977; Silva, 2008), plus supplementary mapping or other analysis. Finally, the report containing the proposed set of considerations is drafted for submission to the final government-level decision-making team, with a view to introducing the execution priorities. In the test study, the reasoning is based on impacts, regional needs and project
FIELD RESULTS

The scope of the problem addressed here corresponds to a country with a population of 200 million, an area of 8.5 million square kilometres, one of the world’s seven largest GDPs (simultaneously facing social and environmental disparities) and a transportation matrix in which highway flows truly predominate (Road = 58%) (PNLT 2007, p.8).

The test-case study involved ranking Brazilian transportation infrastructure projects at a given cross-sectional decision-making point of the pilot study developed by Silva (2008). Its main purpose was to validate the procedure for general use in this type of decision-making.

Application of the International Standard ISO 26000 (ISO 26000 2010) comprises part of the planning effort to offer modal waterway and railway alternatives and effective modal transfer to reduce CO₂ emissions, domestic costs and the ‘Brazil Cost’ while also preserving natural habitats and biodiversity. In that light, investments scheduled until 2023 are directed toward achieving balance in the transportation matrix (Road=33%, Rail=32%, Water=29%, Pipeline=5%, Air=1%), according to PNLT (2007, p.8). Considering the CO₂/TKU emission data by transportation modes in Table 1 (below) and current cargo transportation throughput of the order of 1.028 trillion TKU, there will be a reduction of the order of 55 million metric tons.

In the pilot study, application of the procedure for validation purposes entailed initially conducting semi-structured interviews with 53 stakeholders selected among managers of transportation enterprises of the federal government agency responsible for implementing transportation projects; leaders of transportation and foreign trade sector professional associations; technicians and heads of regulatory agencies and highway and waterway research institutes, in order to define the study preconditions. For this initial stage, the technical team endeavoured to form a very representative group of government departments and agencies and other significant stakeholder groups. In this regard, although transportation impact analysis has previously been restricted to local sound, visual and direct-action pollution (Banister, 2002), today, analysis must be broad and universal, taking into account non-renewable fuels, preservation of natural habitats and biodiversity, and other human health and well-being factors. Accordingly, the list of experts must reflect these concerns. With a view to addressing these considerations, the procedure and its combination of methods must be an effective tool to support policy- and decision-makers in their analyses and priority setting as they make decisions to improve transportation infrastructure projects.

For this same purpose, three types of questionnaire were developed by Silva (2008) and submitted to a carefully selected group of respondents for the second stage of the procedure. This produced key decision criteria and weightings, as well as a list of the most necessary projects. Table 2 shows the resulting criteria and their meanings, with weights reflecting their significance to the respondents (however, without any precise relation with project type or region of Brazil for execution), both statistically treated and as far as compliance with project preconditions was concerned, and the stakeholders’ stated expectations and needs. A more refined list of most urgent projects thus results from rigorous analysis by the respondents and the technical team, and resubmission of the questionnaire in a number of rounds, with a view to reinforcing their perceptions of the overall impact of the choices they make.

In the following stages, the results were ranked by priority (on the basis of mean weights of the decision criteria) and by transportation mode groups (simplified in the test-study), such as those from the TOPSIS and TOPSIS+AHP stages. Criteria weightings for the classic TOPSIS and the TOPSIS-AHP combination are compared in Table 3 (below). The set of respondents differed in the two stages, as a result of increasing involvement in the study and knowledge of regional issues. Here, the respondents tended to be a sub-group about 20% smaller than the initial set, comprising

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Table 1 CO₂ emissions/TKU by mode of transport

<table>
<thead>
<tr>
<th>Mode</th>
<th>CO₂/TKU, in g.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highway</td>
<td>164</td>
</tr>
<tr>
<td>Railway</td>
<td>48.1</td>
</tr>
<tr>
<td>Inland waterway</td>
<td>33.4</td>
</tr>
</tbody>
</table>

Source: EHG-Port of Ennshafen, Austria (Fialho 2010)
Table 2. Decision criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Interpretation (very important to be considered)</th>
<th>Weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermodal connection</td>
<td>Contribution to developing connectivity and multimodality</td>
<td>0.30</td>
</tr>
<tr>
<td>Job creation</td>
<td>Influence on the creation of direct and indirect employment in affected regions</td>
<td>0.15</td>
</tr>
<tr>
<td>Social well-being</td>
<td>Effect in fostering benefits, particularly in education, health, safety and mobility</td>
<td>0.10</td>
</tr>
<tr>
<td>Environmental impact</td>
<td>Interferences in the environment, both adverse and beneficial</td>
<td>0.15</td>
</tr>
<tr>
<td>Recovery of environmental losses</td>
<td>Environmental quality gains, in other locations, resulting from project execution</td>
<td>0.10</td>
</tr>
<tr>
<td>Costs and benefits</td>
<td>Funding required for execution vs. possible resulting benefits</td>
<td>0.10</td>
</tr>
<tr>
<td>Execution timeframe</td>
<td>Time elapsed until benefits fully available</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Source: Silva, 2008

Table 3. Comparison of criteria weights used in Classical TOPSIS and TOPSIS+AHP

<table>
<thead>
<tr>
<th>Criteria</th>
<th>TOPSIS*</th>
<th>TOPSIS+AHP criteria weightings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Road</td>
<td>Rail</td>
</tr>
<tr>
<td>Intermodal connection</td>
<td>0.30</td>
<td>0.18</td>
</tr>
<tr>
<td>Job creation</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>Social well-being</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Environmental impact</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>Recovery of environmental losses</td>
<td>0.10</td>
<td>0.19</td>
</tr>
<tr>
<td>Costs and benefits</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>Execution timeframe</td>
<td>0.10</td>
<td>0.11</td>
</tr>
</tbody>
</table>

* one criterion weighting for all modes
† Bypass or Contour projects
Source: Silva (2008)

Application of classical TOPSIS returned 0.68 as the best priority coefficient value $\varphi$ and 0.13 as the lowest, corresponding to the 60th project. This limit was applied in this study because, from there on, values below 0.50 were considered of little significance.

In the pilot study, in stage four of the procedure, a set of 25 projects was chosen from these 60 projects with significant priority values (those with values of $\varphi$ from 0.50 and up). In this set, responses to a pair-by-pair comparison of criteria, by project, obeyed the table of Saaty (1980), which considers 0.10 as the maximum acceptable inconsistency ratio. That ratio proved to be an excellent yardstick for performing sensitivity analyses to obtain a solution with an acceptable consistency ratio in the pilot study. Once again, in this stage, the number of respondents is substantially lower, because it demands greater precision and more detailed pair-wise comparative evaluation and large numbers of criteria in the priority evaluation matrix method (Saaty 1980) and projects evaluated.

TOPSIS was then reapplied with the weightings by criterion and mode-group obtained with the partial application of AHP to the full list of projects of stage 2. The result was coefficients $\varphi$ (priorities) between 0.51 and 0.34. Applying the outputs from partial AHP application as project weighting inputs made it possible to correct trends introduced by informal criteria adjustments initially made to the classic TOPSIS by respondents and/or the technical team.

DISCUSSION

The pilot study sought to validate the procedure by allaying the precision of AHP with the directness of TOPSIS. Hence the proposal to group projects by type and obtain...
Table 4. Comparison of project rankings

<table>
<thead>
<tr>
<th>Projects</th>
<th>( \varphi_{\text{TOPSIS}} )</th>
<th>Rank</th>
<th>( \varphi_{\text{Mean TOPSIS+AHP}} )</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferronorte railway: construction of Alto Araguaia-Rondonópolis sections</td>
<td>0.68</td>
<td>1</td>
<td>0.50</td>
<td>7</td>
</tr>
<tr>
<td>São Paulo ring railway: north section</td>
<td>0.68</td>
<td>2</td>
<td>0.55</td>
<td>2</td>
</tr>
<tr>
<td>Rio de Janeiro ring road, including BR-101 road</td>
<td>0.64</td>
<td>3</td>
<td>0.57</td>
<td>1</td>
</tr>
<tr>
<td>Tucuruí, PA locks</td>
<td>0.64</td>
<td>4</td>
<td>0.52</td>
<td>4</td>
</tr>
<tr>
<td>Santos road access to port, right and left banks</td>
<td>0.64</td>
<td>5</td>
<td>0.54</td>
<td>3</td>
</tr>
<tr>
<td>BR-163 road, Guarantã do Norte (MT) - Rurópolis/Santarém (PA) section</td>
<td>0.63</td>
<td>6</td>
<td>0.47</td>
<td>10</td>
</tr>
<tr>
<td>Port of São Francisco do Sul (SC), berth construction and refurbishment</td>
<td>0.63</td>
<td>7</td>
<td>0.52</td>
<td>5</td>
</tr>
<tr>
<td>Port of Vila do Conde (PA), expansion</td>
<td>0.63</td>
<td>8</td>
<td>0.50</td>
<td>9</td>
</tr>
<tr>
<td>São Paulo ring road: south section</td>
<td>0.61</td>
<td>9</td>
<td>0.50</td>
<td>8</td>
</tr>
<tr>
<td>Porto of Itaquí (MA) – dredging, expansion, refurbishment</td>
<td>0.59</td>
<td>10</td>
<td>0.33</td>
<td>25</td>
</tr>
<tr>
<td>Norte-Sul railway (TO)</td>
<td>0.59</td>
<td>11</td>
<td>0.42</td>
<td>15</td>
</tr>
<tr>
<td>Rail branch line: section Estreito-Balsas</td>
<td>0.57</td>
<td>12</td>
<td>0.44</td>
<td>13</td>
</tr>
<tr>
<td>Paraná-Paraguai waterway: dredging and rock removal</td>
<td>0.55</td>
<td>13</td>
<td>0.44</td>
<td>12</td>
</tr>
<tr>
<td>Port of Itaguai (RJ), dredging</td>
<td>0.55</td>
<td>14</td>
<td>0.43</td>
<td>14</td>
</tr>
<tr>
<td>BR-101 (ES) road, including Vitória bypass</td>
<td>0.55</td>
<td>15</td>
<td>0.36</td>
<td>22</td>
</tr>
<tr>
<td>Nova Transnordestina railway</td>
<td>0.55</td>
<td>16</td>
<td>0.39</td>
<td>20</td>
</tr>
<tr>
<td>Curitiba ring railway</td>
<td>0.54</td>
<td>17</td>
<td>0.61</td>
<td>6</td>
</tr>
<tr>
<td>BR-470 road expressway to port of Itajai (SC)</td>
<td>0.53</td>
<td>18</td>
<td>0.44</td>
<td>11</td>
</tr>
<tr>
<td>Porto of Suape (PE), road access</td>
<td>0.53</td>
<td>19</td>
<td>0.39</td>
<td>18</td>
</tr>
<tr>
<td>Porto of Parangauá (PR): construction and refurbishment</td>
<td>0.52</td>
<td>20</td>
<td>0.40</td>
<td>17</td>
</tr>
<tr>
<td>Camaçari – Aratú (BA): ring railway</td>
<td>0.52</td>
<td>21</td>
<td>0.41</td>
<td>16</td>
</tr>
<tr>
<td>BR-230 (PA) road section Marabá, Altamira, Medicilândia, Rurópolis</td>
<td>0.52</td>
<td>22</td>
<td>0.39</td>
<td>19</td>
</tr>
<tr>
<td>BR-364 (MT) road: section Diamantino-C.N. dos Parecis</td>
<td>0.52</td>
<td>23</td>
<td>0.35</td>
<td>24</td>
</tr>
<tr>
<td>BR-153 (TO) road: Section GO/TO - TO/PA boundary</td>
<td>0.52</td>
<td>24</td>
<td>0.35</td>
<td>23</td>
</tr>
<tr>
<td>Port of Santos (SP): dredging and rock removal</td>
<td>0.51</td>
<td>25</td>
<td>0.37</td>
<td>21</td>
</tr>
</tbody>
</table>

Source: Silva (2008); Silva et al. (2014)

mean weightings by criteria derived from AHP from the project or transportation-mode groups. The grouped results for the 25 projects revealed major differences on criteria such as inter-modal connection (which had a much greater influence in the classical TOPSIS) and the two direct environmental criteria, whose values denoted significant changes in the weightings (Table 3 above). Table 4 (below) compares the project rankings (rank) by priority, as obtained from the classic TOPSIS (\( \varphi_{\text{TOPSIS}} \)) and by TOPSIS+AHP, inputting the mean criteria weightings by project group (\( \varphi_{\text{Mean}} \)).

The analysis indicates that even classic TOPSIS would be a reasonably adequate decision-analysis tool in certain situations (for instance, where it is difficult to engage experts and stakeholders in detailed, laborious work). This seemed a reasonable option since the 25 best projects, as ranked by classical TOPSIS, figure among the projects with the highest priority rankings resulting from the final stage of the procedure, in the test study outcomes.

From this pilot onwards, by-project complementary analyses were performed and final decisions made to a level of detail not present in this case study. This involved considering other data and assessing cross-impacts and regional priorities. However, at the time of the pilot, the government agency’s executive group validated the final
TOPSIS+AHP ranking as forming a valuable basis for decision-making, coming close to what was actually intended for execution (about 24% of the priority projects are railways).

CONCLUSIONS

The assertive methodology confirmed the suitability of using DELPHI, TOPSIS and AHP jointly, as done in some papers presented in the literature review.

Brainstorming in a selected expert group starts the decision-making process. DELPHI should be emphasised as a complementary step, because of its fitness for this task, which is limited by expert group size constraints and heightened by currently available information technology, which offers the possibility of investigating experts’ opinions without interaction among them. Note also the role of the review that guided to effective and simpler use of ranking methods such as TOPSIS, as well as partial use of AHP, which proved effective to refine the outcomes before the final analysis made in order to report the final outcome to top-level decision-makers. This set of techniques can be used (and was) to set priorities for implementing sustainable transport infrastructure projects in Brazil.

The literature review showed the applicability of some of the more traditional methods, TOPSIS, AHP and DELPHI (including their use in association), which guided consolidation of the structure of the procedure proposed in Figure 1. In addition, the actual case study results indicated that the procedure, using a mixture of multi-criteria and qualitative forecasting methods, meets the challenge of selecting a project sequence for execution by place and time from a high number (over 400, in some cases) of alternatives.

Some of the applications mentioned in this paper have generally been limited to a small number of alternatives and technically-defined criteria. Murphy et al. (1992), Kline (1997) and Mingoti (2013) are not used in this pilot study, but should be in future work. Meanwhile, the simplicity of TOPSIS facilitates participatory planning, despite its acknowledged weakness, which is the use of the same weighting scale for all options evaluated. However, in its application to the groups of projects first selected (DELPHI stage), the ranking generated might suffer from the weakness associated with the first table of weightings, but this is neutralised by the AHP pair-by-pair comparative analysis of the criteria and the new calculus of criteria weightings by project-type. Thus comparing the criteria and their relative importance calls for careful work by experts. In fact, applying the AHP methodology successfully demands some expertise and knowledge of the problem at hand, as well as attaining the desired level of consistency (Saaty 1980; 1991), often calling for extra adjustment and orientation until a significant partial result enables the analysis to proceed, besides the requirement for there to be no correlation among the criteria. Mathematical resources such as triangular fuzzy number logic and multi-criteria approaches are certainly a step forward for such evaluation.

The main goal of the test-case study was to validate the procedure in a real context. For that reason, reinforced by the fact that partial and final results of a pilot study (with limitations relevant to this study) had been presented, in order to validate the procedure, the selection of only 25 projects for application of AHP, and the bound of 7 impact attributes, was closely related to the goal, which was to measure the cross-impact on the optimal sets of attributes, which were not yet accurately differentiated by the relative importance of each pair in terms of the study preconditions. In other possible analyses, as discussed in Silva et al. (2014), the desired balance in the transportation matrix for 2023 – and consequently the eco-sustainability of the projects by using green energy sources – is to be achieved through a multimodal network in which interurban circuits (with high-speed train components) will integrate with regional medium-speed networks in connection with the (refurbished) existing network, not just for cargo, goods and valuables, but for passenger services as well. Already, the criteria established in this study permitted an analysis suggesting that sizable investments in the cargo and passenger rail transportation system would stimulate employment generation (job creation) and intermodal connections, in urban centres as well. As Silva & Cavalcanti Netto (2012) pointed out, ‘long-term results can only be accomplished by enacting industrial, trade and agricultural policies to support broad-minded national strategic goals and by pursuing ethical preservation of environmental biodiversity’.

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Appendix

(A) Some maths for TOPSIS application (based on Hwang & Yoon 1981)

1st Step:

A preliminary assembly of the array in which \( j \) alternatives (projects) are represented by \( a_1, a_2, a_3, \ldots, a_J \). For each alternative, an assessment of each project on each decision-making criterion of order \( i \) is represented by \( a_{ij} \), where \( N \) is the number of criteria and \( J \) is the number of projects evaluated. The main array is:

\[
A = [a_{ij}] = \\
\begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1j} & \ldots & a_{1N} \\
a_{21} & a_{22} & \ldots & a_{2j} & \ldots & a_{2N} \\
a_{31} & a_{32} & \ldots & a_{3j} & \ldots & a_{3N} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{ij} & a_{2j} & \ldots & a_{ij} & \ldots & a_{NJ} \\
\end{bmatrix}
\]

2nd Step:

For a normalised matrix, two approaches – linear or by vector – are used. In the linear approach, the calculation can be done, among other ways, by determining the ratio \( r_{ij} \) between the value assigned to a project criterion \( a_{ij} \) and the maximum value assigned to each project by criterion. The calculation is performed using equation (1) below:

\[
r_{ij} = \frac{a_{ij}}{a_j^{*}} \quad i = 1, \ldots, N; \quad j = 1, \ldots, J \quad (1),
\]

Where \( a_j^{*} \) is the maximum \( j \) criterion. Thus \( 0 \leq r_{ij} \leq 1 \), with 1 the most favourable.

A normalisation vector can be calculated using equation (2) below:

\[
r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{j=1}^{J} a_{ij}^2}} \quad i = 1, 2, 3, \ldots, N; \quad j = 1, \ldots, J \quad (2),
\]

and the next matrix, \( A_n \), corresponds to the normalised matrix \( A \) with the values of \( a_{ij} \) replaced by \( r_{ij} \):

\[
A_n = [r_{ij}] = \\
\begin{bmatrix}
r_{11} & r_{21} & r_{31} & \ldots & r_{1j} & \ldots & r_{1N} \\
r_{12} & r_{22} & r_{32} & \ldots & r_{2j} & \ldots & r_{2N} \\
r_{13} & r_{23} & r_{33} & \ldots & r_{3j} & \ldots & r_{3N} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
r_{ij} & r_{2j} & r_{3j} & \ldots & r_{ij} & \ldots & r_{NJ} \\
\end{bmatrix}
\]

3rd Step:

Calculation of a weighted matrix consisting of weighted values \( v_{ij} \), obtained by multiplying each value \( r_{ij} \) by the weight \( w_i \) assigned to each of the \( n \) criteria of equation (3), which were obtained by other methods.

\[
v_{ij} = r_{ij} \times w_i \quad i = 1, 2, 3, \ldots, N; \quad j = 1, \ldots, J \quad (3),
\]

under the condition \( \sum_{i=1}^{n} w_i = 1 \).
4th Step:

Determination of two solutions, an ideal positive ($A^+$) and an ideal negative ($A^-$), the former represented by each maximum value of weighted criterion and the latter by the minimum value, calculated the same way as in equations (4) and (5) below:

\[
A^+ = \{ v_i^+, \ldots v_i^+ \} = \{ \max_j v_{ij} | i \in I' \}, \quad \{ \min_j v_{ij} | i \in I'' \} \quad \text{(4), and} \\
A^- = \{ v_i^-, \ldots v_i^- \} = \{ \min_j v_{ij} | i \in I' \}, \quad \{ \max_j v_{ij} | i \in I'' \} \quad \text{(5),}
\]

$I'$ being associated with the criterion of benefit and $I''$ with the cost criterion.

5th Step:

Calculation of deviations from Euclidean distance between the weighted values for each alternative $j$, as in equations (6) and (7) below:

\[
\Delta_j^+ = \sqrt{\sum_{i=1}^{n} (v_{ij} - v_i^+)^2} \quad j = 1, 2, 3, \ldots J \quad \text{(6)}, \\
\Delta_j^- = \sqrt{\sum_{i=1}^{n} (v_{ij} - v_i^-)^2} \quad j = 1, 2, 3, \ldots J \quad \text{(7)},
\]

6th Step:

Determining the outcome of each alternative relative to ideal positive and negative situations, represented by $\varphi$.

Finally the result of the approach to positive and negative situations is reached using equation (8) below:

\[
\varphi = \frac{\Delta_j^-}{(\Delta_j^+ + \Delta_j^-)} \quad j = 1, 2, 3, \ldots J \quad \text{(8)}
\]

The highest value of $\varphi$ corresponds to the best alternative within the assessment principles used. MS-Excel™ offers good results for solving TOPSIS algorithm with flexibility and speed to provide good conditions for sensitivity analysis.

Application of TOPSIS as shown in equation 8 returned the priority coefficients $\varphi$, defined as the ratio of the negative deviation $\Delta^-$ to the sum of the deviations ($\Delta^+ + \Delta^-$), where $\Delta^+$ is the deviation in relation to the least ideal situation (the worst mean in the project/criterion correlation) and $\Delta^-$ is the deviation in relation to the best project/criterion relationship. The larger the $\Delta^-$, the more favourable the evaluation; and vice versa for $\Delta^+$.

(B) The Partial AHP for re-calculating criteria weightings (with more precision) (based on Saaty 1980; 1991; 2011)

According to Saaty (1980; 1991), the AHP method enables a vector of priorities or criteria weightings to be defined for each project. Thus, it defines a square matrix $A$ (below) of crossing weights by $A = (a_{ij}), (i, j = 1, 2, 3, \ldots n)$ with the rules for the definition of $a_{ij}$:

firstly, if $a_{ij} = \alpha$, then $a_{ji} = \frac{1}{\alpha}$. Secondly, if $C_i$, for instance, is deemed of equal importance to $C_j$, it is assumed that $a_{ij} = 1$; and in the specific situation that $a_{ii} = 1$, for any $i$ (Saaty 1980). The evaluation could be made by comparing pairs of
alternatives or attributes, based on a value scale for degree of importance from 1 (low) to 9 (high), depending on the goal under review. Thus, the basic AHP matrix includes the reciprocal values (matrix \( A \)). Criteria are compared by double-entry filling square matrices. The criteria are repeated in rows and columns, and the intersection shows the result. Table A (below) reproduces the scale of Saaty (1991; 2011), which establishes the degree of importance when comparing attributes or criteria.

\[
A = \begin{bmatrix}
1 & a_{12} & a_{13} & \cdots & a_{1n} \\
1 & 1 & a_{22} & \cdots & a_{2n} \\
1 & 1 & 1 & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \cdots & 1
\end{bmatrix}
\]

It can be seen that \( 1/a_{12} \) is the reciprocal of \( a_{12} \), i.e., if the analyst finds that the attribute \( a_{21} \) is five times larger than \( a_{12} \), the reciprocal will be equal to 0.2, and so on.

The next step of AHP is the normalisation of the matrix. Saaty (1991) proposes different manners of doing this. He states that they all yield similar results, particularly if the matrix has consistent data. One of the procedures is to construct a new normalisation matrix \( A_y \), where the elements of each column \( (r_{ij}) \) result from dividing data from the original column by the sum of the same column as shown in equation 9:

\[
r_{ij} = \frac{a_{ij}}{\sum_{j=1}^{n} a_{ij}} \quad \text{where } i = 1, 2, 3, \ldots, n
\]

After that, we obtain the vector of priorities (or weights) \( P \) (formed by \( P_i \)) by calculating all the average of the normalised rows of the matrix \( A_y \) (equation 10).

\[
P_i = \left( \frac{1}{n} \right) \sum_{j=1}^{n} r_{ij} \quad \text{where } i = 1, 2, 3, \ldots, n
\]

The next round of the AHP method (‘synthesis’) is to determine the consistency of evaluations according to a corresponding index obtained from the calculation of eigenvalue \( \lambda_{\text{max}} \), multiplying the matrix formed by the sum of the columns of the matrix \( A \) in the form \( V^T \) by the vector of priorities (\( P \)), as recommended by Saaty (1980) (equation 11):

\[
\lambda_{\text{max}} = V^T \cdot P
\]

based on eigenvalue \( \lambda_{\text{max}} \), the next step determines the consistency index (\( IC \)), also called deviation consistency:

\[
IC = \frac{(\lambda_{\text{max}} - n)}{(n - 1)}
\]
Table A Fundamental scale of absolute numbers

<table>
<thead>
<tr>
<th>Definition</th>
<th>Explanation</th>
<th>Intensity of importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective</td>
<td>1</td>
</tr>
<tr>
<td>Weak or slight</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Moderate importance</td>
<td>Experience and judgment slightly favour one activity over another</td>
<td>3</td>
</tr>
<tr>
<td>Moderate plus</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>Strong importance</td>
<td>Experience and judgment strongly favour one activity over another</td>
<td>5</td>
</tr>
<tr>
<td>Strong plus</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>Very strong or demonstrated</td>
<td>An activity is favoured very strongly over another; its dominance is demonstrated in practice</td>
<td>7</td>
</tr>
<tr>
<td>Very, very strong</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extreme importance</td>
<td>The evidence favouring one activity over another is of the highest possible order of affirmation</td>
<td>9</td>
</tr>
<tr>
<td>A better alternative way to assign small decimals is to compare two close activities with other widely contrasting ones, favouring the larger one a little over the smaller one when using the 1-9 values.</td>
<td>1.1 – 1.9</td>
<td></td>
</tr>
<tr>
<td>A logical assumption</td>
<td>If activity (i) has one of the above non-zero numbers assigned to it when compared with activity (j), then (j) has the reciprocal value when compared with (i)</td>
<td></td>
</tr>
<tr>
<td>When it is desired to use such numbers in physical applications. Alternatively, often one estimates the ratios of such magnitudes by using judgment</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Saaty (1991; 2011)

Table B Random coefficients in relation to the size of the matrices

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(IR)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

Source: Saaty (1991)

The final part of the AHP synthesis consists in determining a consistency ratio (\(RC\)) obtained from experiments performed with data from \(IC\) at Oak Ridge National Laboratory (USA) and the University of Wharton. Simulations with 9x9 and 11x11 matrices were generated randomly, with reciprocal forced Random Index (\(IR\)), a kind of random consistency. Depending on the results obtained with 100 samples (Oak Ridge) and 500 (Wharton), Saaty (1991) formulated Table B (above), which shows the values of \(RC\) in relation to the order \(n\) of the matrices.

Thus, the consistency ratio \(RC\) is an indicator showing the actual consistency of the responses obtained through research and AHP is determined by:

\[
RC = \frac{IC}{IR}
\]  

(13)

To Saaty (1991), \(RC \leq 0.10\) means that valuations have produced good quality and are considered acceptable.
(C) A flowchart for traditional DELPHI method application (Figure A, based on Sullivan & Claycombe 1977)

Fig. A - Flowchart of the DELPHI method