Review

On nonlinear decoupling-controller design for maglev vehicles

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To enhance the reliability of levitation, an especial installation called joint-structure is applied to the maglev vehicle. Due to the joint-structure, each suspension point of the maglev vehicle is regulated by two independent controllers. When one controller of the suspension point breaks down, the system can still be stably suspended by the other controller. However, there is strong force-coupling between the two controllers for the joint-structure, which makes the controller designed based on totally separated suspension point unsuccessful in application. To realize stable suspension for joint-structure, nonlinear decoupling control techniques is introduced to obtain a globally decoupling and linearized model for the system. Then the control parameters are designed by pole assignment. Experimental and simulation results validate the effectiveness of the proposed control algorithm.

Keywords: Maglev vehicles, Nonlinear decoupling control, Global stability.

INTRODUCTION

Maglev technology has been widely utilized to different applications (Joo and Seo 1997), among which maglev vehicles rapidly develops since its presentation. To enhance the reliability of maglev vehicles, joint-structure is introduced, and each joint-structure is regulated by two controllers. If one controller breaks down, the other controller can still guarantee the stability of the system, and the stability of the vehicle accordingly. As the two controllers for the same joint-structure is strongly coupled together, decoupling control is the key to the stability of the joint-structure. There are quite a lot of decoupling control strategies, such as inversion matrix method (Morales et al., 2011; Ulbig et al., 2010), relative amplification coefficient matching method (Shiskoy FG 1977), diagonal dominance approach (Rosebrocok HH 1969), state feedback control (Lingling et al., 2012; Zhang et al., 2010),adaptive decoupling control (Zi-Jiang and Michitaka., 2001), intelligent decoupling control (Morari M 1994; Shaw and Doyle 1997), nonlinear and robust decoupling control (eza et al., 2012; Lingling et al., 2011), and so on.

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Based on the characteristics of the maglev system and the experimental setup, this article uses differential geometry method to realize the decoupling and the stability of the joint-structure.

Modeling

Ignoring the elasticity of the guideway and the effect of the air-spring on the system performance, the sketch of the joint-structure can be presented by Figure 1 (Shen et al., 2008).

The definition of the symbols in Figure 1 and the symbols which will be used in the following are presented as below.

$s_{01} = s_{02} = s_0$ is the expected suspension gap, s_1 and s_2 is the measured suspension gap, $m_1 = m_2 = m$ is mass of the suspension object, g is the Gravity acceleration, F_1 and F_2 is the electromagnetic force, u_1 and u_2 is the control voltage, i_1 and i_2 is the current in the coil, $N_1 = N_2 = N$ is the turns of the coil, $R_1 = R_2 = R$ is the resistance of the coil, $A_1 = A_2 = A$ is the pole area of a single side of the electromagnet, L_1 and L_2 is the inductance of the coil, μ_0 is the permeability of vacuum, k_r is the rigid coefficient between two sides.

Suppose the magnetomotive force of the magnetic field concentrates in the air gap, the mathematical model of the system shown in Figure 1 can be obtained as (Shen et al., 2008),

$$\begin{cases} u_1 = Ri_1 + \dfrac{\mu_0 N^2 A}{2s_1}\dfrac{di_1}{dt} - \dfrac{\mu_0 N^2 A i_1}{2s_1^2}\dfrac{ds_1}{dt} \\ m\ddot{s}_1 = -F_1(i_1, s_1) + mg - k_r(s_1 - s_2) \\ u_2 = Ri_2 + \dfrac{\mu_0 N^2 A}{2s_2}\dfrac{di_2}{dt} - \dfrac{\mu_0 N^2 A i_2}{2s_2^2}\dfrac{ds_2}{dt} \\ m\ddot{s}_2 = -F_2(i_2, s_2) + mg + k_r(s_1 - s_2) \end{cases} \quad (1)$$

$$F_i(i_i, s_i) = \left(\mu_0 N^2 A/4\right)\left(i_i/s_i\right)^2$$

In (1), the first and the second equations are separately the electricity equation and mechanical equation for the suspension system on the left, while the third and the fourth equations are separately the electricity equation and mechanical equation for the suspension system on the right.

Decoupling control theory

For the convenience of problem analysis, the decoupling control theory used in this paper is listed as following (XiaoHua and Weibing 1993).

Definition 1: Let open sets, $U \subset R^n$, $x \subset U$, and given

a smooth scalar function $\lambda(x)$ and a n-dimention vector field $f(x)$ in U. Then a new scalar function noted $L_f\lambda(x)$ is defined as following.

$$L_f\lambda(x) = \sum_{i=1}^{n}\frac{\partial\lambda}{\partial x_i}f_i(x_1, x_2, ..., x_n) \quad (2)$$

This new function $L_f\lambda(x)$ is called the lie-algebra of $\lambda(x)$ along $f(x)$. And this function can be calculated iteratively. For example, lie-algebra of $\lambda(x)$ sequentially along $f(x)$ and $g(x)$ is,

$$L_g L_f\lambda(x) = \frac{\partial(L_f\lambda)}{\partial x}g(x) \quad (3)$$

Or lie-algebra of $\lambda(x)$ along $f(x)$ for k times is,

$$L_f^k\lambda(x) = d(L_f^{k-1}\lambda)f(x) \quad (4)$$

Definition 2: For the following multiple inputs and multiple outputs system,

$$\dot{x} = f(x) + \sum_{i=1}^{m}g_i(x)u_i \quad (5)$$

$$y_j = h_j(x), j \in \underline{m}$$

Its relative order $r_i(x_0)$ is the lie algebra, which satisfies,

$$L_{g_j}L_f^{r_i-1}h_i(x_0) \neq 0(j = 1, ..., m)$$

And for $k < r_i - 1$, satisfies,

$$L_{g_j}L_f^k h_i(x_0) = 0(j = 1, ..., m)$$

On the basis of the two definitions, two theorems for decoupling of nonlinear systems are introduced. First, introduce the decoupling matrix nonsingular theorem.

Theorem 1: If a multiple variables nonlinear system has a relative order r_i for x_0 to all $i \in \underline{m}$, its decoupling matrix $A(x)$ shown in (6) is nonsingular at x_0.

$$\begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(x) & \cdots & L_{g_m}L_f^{r_1-1}h_1(x) \\ L_{g_1}L_f^{r_2-1}h_2(x) & \cdots & L_{g_m}L_f^{r_2-1}h_2(x) \\ \vdots & \cdots & \vdots \\ L_{g_1}L_f^{r_m-1}h_m(x) & \cdots & L_{g_m}L_f^{r_m-1}h_m(x) \end{bmatrix} \quad (6)$$

Then introduce theorem 2, which is utilized to explain the conditions for decoupling of the system, and how to realize decoupling for nonlinear coupled system.

Theorem 2: If nonlinear system has relative order at x_0, or saying decoupling matrix $A(x)$ is nonsingular at x_0, then the input-output decoupling problem at x_0 can be solved by a static state feedback, and one solution is the feedback defined by the following matrix,
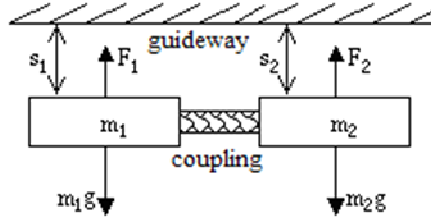
Figure1: Sketch of joint-structure

$$u(x) = \alpha(x) + \beta(x)v$$
$$\alpha(x) = -A^{-1}(x)b(x) \qquad (7)$$
$$\beta(x) = A^{-1}(x)$$

where,

$$b(x) = \begin{bmatrix} L_f^{r_1} h_1(x) & \cdots & L_f^{r_m} h_m(x) \end{bmatrix}^T \qquad (8)$$

If the conditions described in theorem 2 are satisfied, decoupling for nonlinear system can be realized by equations (7) and (8). The proposed nonlinear decoupling method for maglev system can be obtained based on the definitions and theorems introduced in this part.

Decoupling and linearization of the system

For the convenience of description, introduce the following variables transform,

$$\begin{aligned} x_1 = s_1 \quad x_2 = y_1 \quad x_3 = i_1 \\ x_4 = s_2 \quad x_5 = y_2 \quad x_6 = i_2 \end{aligned} \qquad (9)$$

$$k = \mu_0 N^2 A/4 \qquad (10)$$

By the transform in (9), the model (1) can be changed as following,

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = g - \dfrac{kx_3^2}{mx_1^2} - \dfrac{k_r(x_1 - x_4)}{m} \\ \dot{x}_3 = \dfrac{x_2 x_3}{x_1} - \dfrac{Rx_1 x_3}{2k} + \dfrac{x_1}{2k}u_1 \\ \dot{x}_4 = x_5 \\ \dot{x}_5 = g - \dfrac{kx_6^2}{mx_4^2} + \dfrac{k_r(x_1 - x_4)}{m} \\ \dot{x}_6 = \dfrac{x_5 x_6}{x_4} - \dfrac{Rx_4 x_6}{2k} + \dfrac{x_4}{2k}u_2 \end{cases} \qquad (11)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} X \qquad (12)$$

Compared with the standard form in (5), it can be known:

$$f(x) = \begin{bmatrix} x_2 \\ g - kx_3^2/mx_1^2 - k_r(x_1 - x_4)/m \\ x_2 x_3/x_1 - Rx_1 x_3/2k \\ x_5 \\ g - kx_6^2/mx_4^2 + k_r(x_1 - x_4)/m \\ x_5 x_6/x_4 - Rx_4 x_6/2k \end{bmatrix} \qquad (13)$$

$$g_1(x) = \begin{bmatrix} 0 & 0 & x_1/2k & 0 & 0 & 0 \end{bmatrix}^T \qquad (14)$$

$$g_2(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & x_4/2k \end{bmatrix}^T \qquad (15)$$

$$y_1 = h_1(x) = x_1 \qquad (16)$$
$$y_2 = h_2(x) = x_4 \qquad (17)$$

First, the decoupling matrix of the system can be calculated according to (6) as,

$$A(x) = \begin{bmatrix} -x_3/mx_1 & 0 \\ 0 & -x_6/mx_4 \end{bmatrix} \qquad (18)$$

And the other expressions to calculated the control variables can also be obtained as,

$$b(x) = \begin{bmatrix} b_1 & b_2 \end{bmatrix}^T =$$
$$\begin{bmatrix} \dfrac{Rx_3^2}{mx_1} - \dfrac{k_r}{m}(x_2 - x_5) & \dfrac{Rx_6^2}{mx_4} + \dfrac{k_r}{m}(x_2 - x_5) \end{bmatrix} \qquad (19)$$

$$\alpha(x) = \begin{bmatrix} Rx_3 - k_r x_1(x_2 - x_5)/x_3 \\ Rx_6 + k_r x_4(x_2 - x_5)/x_6 \end{bmatrix} \qquad (20)$$

$$\beta(x) = \begin{bmatrix} -mx_1/x_3 & 0 \\ 0 & -mx_4/x_6 \end{bmatrix} \qquad (21)$$

By equation (7), the decoupling control law can be calculated as,

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} Rx_3 - k_r x_1(x_2 - x_5)/x_3 \\ Rx_6 + k_r x_4(x_2 - x_5)/x_6 \end{bmatrix}$$
$$+ \begin{bmatrix} -mx_1/x_3 & 0 \\ 0 & -mx_4/x_6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad (22)$$

That's to say,

$$u_1 = Rx_3 - \dfrac{k_r x_1}{x_3}(x_2 - x_5) - \dfrac{mx_1}{x_3}v_1 \qquad (23)$$

$$u_2 = Rx_6 + \frac{k_r x_4}{x_6}(x_2 - x_5) - \frac{mx_4}{x_6}v_2 \qquad (24)$$

Up to now, the system has been decoupled. To realize linearization of the decoupled system, the following variable transform is introduced,

$$z_1 = x_1, \qquad z_2 = x_2, \qquad (25)$$

$$z_3 = \dot{z}_2 = g - \frac{kx_3^2}{mx_1^2} - \frac{k_r(x_1 - x_4)}{m} \qquad (26)$$

$$z_4 = x_4, \qquad z_5 = x_5, \qquad (27)$$

$$z_6 = \dot{z}_5 = g - \frac{kx_6^2}{mx_4^2} + \frac{k_r(x_1 - x_4)}{m} \qquad (28)$$

It can be obtained by calculation that,

$$\dot{z}_3 = v_1, \qquad \dot{z}_6 = v_2 \qquad (29)$$

Now, the model for joint-structure has been decoupled and linearized, and the state space model after decoupling is,

$$\dot{z} = Az + Bv \qquad (30)$$

$$y = Cz \qquad (31)$$

where,

$$z = \begin{bmatrix} z_1 & z_2 & \cdots & z_6 \end{bmatrix}^T$$

And the state matrix for the system is,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (32)$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T \qquad (33)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (34)$$

From state space matrix (32)~(34), the state space model for two separated subsystems can be obtained, and the state space for the first subsystem is,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad (35)$$

$$B_1 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T \qquad (36)$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (37)$$

It can be seen from (32)~(34) that, the state matrix of the second subsystem is the same to the first one. Thus only the controller design for the first subsystem is conducted.

Control algorithm design

In this section, control algorithm design refers to the controller design for subsystem (35)~(37). For the convenience of implementation, state feedback control is utilized. Suppose the feedback control law is (Ling W 1990).

$$v_1 = k_1(z_1 - z_{01}) + k_2 z_2 + k_3 z_3 \qquad (38)$$

And set the control goals of the system after feedback as, overshoot is less than 5%, settling time is $0.1\ s$. Then the dominant poles for the system is calculated as,

$$s_1 = -20 + 42.925 j \qquad (39)$$

$$s_2 = -20 - 42.925 j \qquad (40)$$

Suppose the third pole of the system is,

$$s_3 = -200 \qquad (41)$$

Then the characteristic equation for the system is,

$$s^3 + 280s^2 + 19358s + 671542 = 0 \quad (42)$$

Substitute feedback control law (38) into the first subsystem after linearization (35)~(37), and the characteristic equation for the closed loop system can be obtained as,

$$s^3 - k_3 s^2 - k_2 s - k_1 = 0 \qquad (43)$$

Comparing the characteristic equation after feedback control (43) with the expected characteristic equation (42), the control parameters can be chosen as,

$$k_1 = -671542 \qquad (44)$$

$$k_2 = -19358 \qquad (45)$$

$$k_3 = -280 \qquad (46)$$

Substitute (25) and (26) into (38), the control variable for the linear system can be calculated as,
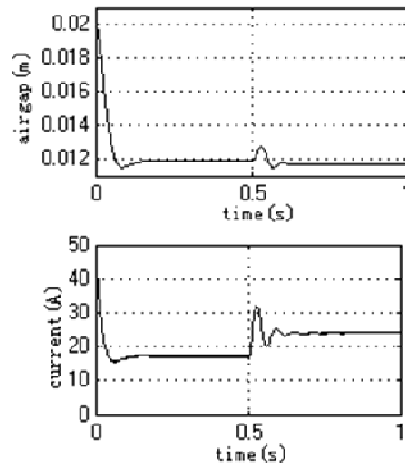
Figure2: Simulation result of air-gap and current

$$v_1 = k_1(x_1 - s_0) + k_2 x_2 +$$
$$k_3 \left(g - \frac{kx_3^2}{x_1^2} - \frac{k_r(x_1 - x_4)}{m} \right) \quad (47)$$

Substitute control law for linear system (47) into the expression of control variable (23), the control variable of the first subsystem for the joint-structure can be obtained as,

$$u_1 = Rx_3 - \frac{k_r x_1}{x_3}(x_2 - x_5)$$
$$-\frac{mx_1}{x_3}\left[k_1(x_1 - s_0) + k_2 x_2 + k_3\left(g - \frac{kx_3^2}{mx_1^2} - \frac{k_r(x_1 - x_4)}{m} \right) \right] \quad (48)$$

Similarly, the control variable for the second subsystem can be calculated as,

$$u_2 = Rx_6 + \frac{k_r x_4}{x_6}(x_2 - x_5)$$
$$-\frac{mx_4}{x_6}\left[k_1(x_4 - s_0) + k_2 x_5 + k_3\left(g - \frac{kx_6^2}{mx_4^2} + \frac{k_r(x_1 - x_4)}{m} \right) \right] \quad (49)$$

Equations (48) and (49) are the control law applied to the physical controllers.

SIMULATION AND EXPERIMENTS

For one application system, the parameters of the two subsystems described in Figure 1 are totally the same.

$k = 0.00545$, $m = 725kg$

$s_0 = 0.012m$, $R = 4.44\Omega$

$g = 9.8N/kg$, $k_r = 1.236 \times 10^7 N/m$

When the power of one controller of the joint-structure is suddenly shut down, the simulation results of the other controller are shown in Figure 2.

When the power of one controller of the joint-structure is suddenly shut down, the experimental results of the other controller are shown in Figure 3.

From simulation results shown in Figure 2, it can be seen that when one controller breaks down, the current for the other controller increase rapidly from 18A to 24A, and the maximum variety of the suspension gap is 0.5mm.

From experimental results shown in Figure 3, it can be seen that when one controller breaks down, the current for the other controller increase rapidly from 17A to 25A, and the maximum variety of the suspension gap is 1.6mm. And the suspension gap finally reaches the expected value because of the effect of integrator.

CONCLUSIONS

The nonlinear model for joint-structure of maglev vehicles are built in this paper. And the obtained model is a multiple-inputs multiple-outputs coupling model. Nonlinear state feedback technique is introduced to decouple and linearize this model. Then pole assignment is utilized to design the controller. The proposed control algorithm is simple, and convenient to implement.
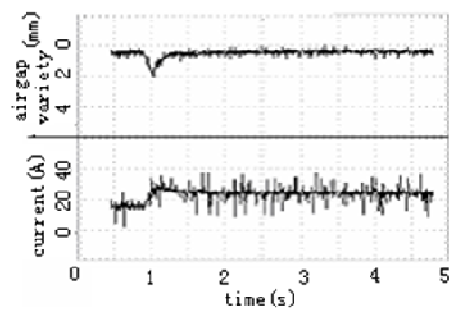
Figure3: Experiment results of air-gap variety and current

The control strategy proposed in this paper is validated by both simulation and experiment. Simulation and experimental results show that, both the two controllers for a single joint-structure can realize stable suspension, which means the proposed method is effective for decoupling control. And it can be also shown that, when one controller breaks down, the other controller can realize the stability of the whole joint-structure system by itself. Joint-structure function is effectively realized by the proposed method, which can improve the safety of maglev vehicles.

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